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TEMPEST Level-0 Theory

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14. ABSTRACT
TEMPEST is a computational tool that is being developed to predict the seakeeping and dynamic stability performance of a steered ship in large waves. The theory for TEMPEST is being developed in several "levels". The difference between the theory levels is in the fidelity and complexity of the environment definition and the component force models. This report describes the "Level-0" theory. The objective of the Level-0 version of TEMPEST is to establish a working computational tool that provides reasonable seakeeping predictions and to establish the framework of the TEMPEST tool into which the more accurate Level II and Level III models can be implemented. The Level-0 theory is based on simple models that include a linear hydrodynamic force model based on user-supplied added mass and damping coefficients, a nonlinear model for the hydrostatic and Froude-Krylov forces, and empirical models for the maneuvering forces and the forces from the bilge keels, rudders and propeller. This report is a compilation of the equations of motion, wave environment, and component-force theory white papers provided to the TEMPEST code developers, DRS Defense Solutions, for implementation of the Level-0 theory.

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Abstract

Tempest is a computational tool that is being developed to predict the seakeeping and dynamic stability performance of a steered ship in large waves. The theory for Tempest is being developed in several "levels". The difference between the theory levels is in the fidelity and complexity of the environment definition and the component force models. This report describes the Level-0 theory. The objective of the Level-0 version of Tempest is to establish a working computational tool that provides reasonable seakeeping predictions and to establish the framework of the Tempest tool into which the more accurate Level-II and Level-III models can be implemented. The Level-0 theory is based on simple models that include a linear hydrodynamic force model based on user-supplied added mass and damping coefficients, a nonlinear model for the hydrostatic and Froude-Krylov forces, and empirical models for the maneuvering forces and the forces from the bilge keels, rudders and propeller. This report is a compilation of the equations of motion, wave environment, and component-force theory white papers provided to the Tempest code developers, DRS Defense Solutions, for implementation of the Level-0 theory.

Administrative Information

The work described in this report was performed at the Naval Surface Warfare Center, Carderock Division (NSWCCD) by the Seakeeping Division, Code 5500. The development of the Tempest Level-0 theory white papers described within this report was funded by Naval Sea Systems Command (NAVSEA), PMS500, under Program Element 0604300N. The work was performed predominately under work unit 07-1-5500-752-10. The writing of this report was funded by NAVSEA 05D1 under the CPSD SE/TA program supporting Tempest development. The associated Program Element is 0603563N and work unit is 09-1-2124-206.

Introduction

Tempest is a computationally efficient time domain tool designed to predict the motions of a steered ship in large waves. It is intended for use as both a seakeeping and a dynamic stability prediction tool. The planning for the development of Tempest was initiated in 2006. The plan called for a multi-phased effort that would promote early progress on the code development while the theory is in development, with the goal being that a more polished and complete program would be available sooner. The first phase of the code development is focused on creating the foundational code across all three program tiers (Client, Business, and Data) and is therefore not expected to be used for actual simulation work applied to real ship designs. However, while there is not a need to have accurate simulation results, the components of the ship motion theory need to be included to the greatest extent possible in order to provide a meaningful basis for developing code related to the inputs and outputs of the Client tier, the interactions of the integrators, force models, and other foundational aspects in the Business tier, and finally the persistence capabilities and needs in the Data tier. To support this need, a so-called “Level-0” theory has been established that provides a nearly complete, though lower fidelity, ship motion theory to be implemented in the first release of Tempest. The Level-0 code is mainly based on simple theories that can be used to develop a working code that produces “reasonable” predictions.

Several “levels” of theory will be developed for Tempest. All levels of the theory are based on the same rigid body equations of motion and time stepping technique. The difference between the levels is in the fidelity and complexity of the environment definition and the component force models. This theory manual describes the theory implemented in “Level-0” Tempest code. The objective of the “Level-0” theory is to develop a well-documented framework into which the more accurate Level-II and Level-III theories can later be implemented. The Level-0 theory will use very approximate methods for certain force components such as the radiation, diffraction and maneuvering force models, as these will be replaced by completely new force models as the Level-II and Level-III theories are developed. Certain components of the Level-0 theory such as the equations of motion and time integration schemes and the hydrostatic and Froude-Krylov force models are intended to carry over to Level-II and possibly even into Level-III with only minor modifications.

The theory described in this document was developed from a variety of sources. The outline of the theoretical structure of Tempest was developed by a core group at Naval Surface Warfare Center, Carderock Division (NSWCCD). A Theory Advisory Panel (TAP) was formed to review and assist in the theory development for Tempest. A series of white papers were written, with each white paper discussing a specific aspect of the code.

The Level-0 theory has been developed in the Seakeeping Division (Code 55) of NSWCCD in West Bethesda, MD. Code development has been performed by DRS Defense Solutions in Stevensville, MD. The theory has been documented and communicated to the code developers incrementally in the form of “Level-0 theory white papers”.

The present document provides a record of these theory white papers within a single report that is able to be referenced. The following document is not intended to be a comprehensive theory manual, but rather a compilation of the theory documents from which the Level-0 code is developed.

1 Nomenclature

The nomenclature tables below describe the variables used in the Tempest theory development. As this is a theory manual that has been developed from the compilation of several component theory documents, the nomenclature has been broken into several sections corresponding to the significant theory areas. The first section describes the nomenclature pertaining to the wind and wave environment. The second section describes the nomenclature used for the general equations of motions and the coordinate systems used throughout the theory. The remaining sections describe the nomenclature used in the various force modules.

The nomenclature will be consistent throughout the theory manual. In some instances a symbol may have multiple definitions. For instance the symbol "*T*" may mean wave period in the wave and wind environment theory, thrust in the propeller force module theory, and draft in the bilge keel damping theory. In these instances it will always be obvious which meaning the symbol has in any given equation. Where possible these conflicts are avoided by not using the same symbol for different variables, but only to the extent to which it helps to clarify the theory. If a commonly accepted symbol exists (i.e., T for both period and draft, λ for wavelength, etc.) then that symbol is always used.

Careful attention is paid, however, to make sure that two different symbols are not used for the same variable (i.e. using both ζ and η for wave elevation in different sections of the theory manual). With regard to the coordinate system definitions, upper case (X,Y,Z) are used to refer to coordinates in the earth-fixed system and lower case (x,y,z) are used to refer to coordinates in the ship-fixed system.

1.1 Wave Environment

Symbol	Definition	SI Units
a_j	(Real-valued) first order wave amplitude equal to half the distance from the trough to the crest	m
a_{jk}	(Real-valued) second order wave amplitude	m
c	Wave celerity (phase velocity)	m/s
c_g	Group velocity of wave	m/s
$E(\,)$	Expected value of a random variable	
f	Wave frequency in units of cycles / second	cycles/s
g	Acceleration due to gravity	m/s^2
G	Directional spreading function	
h	Water depth	m
H_{S}	Significant wave height	m

k	Wave number	rad/m
m_n	n^{th} moment of a spectrum	$m^2(1/sec)^n$
N	Number of wave components	
N_β	Number of spreading angles	
S	Spectral density	m^2sec/rad
t	Time	s
T	Wave period	s
T_m	Modal wave period	s
u_w, v_w, w_w	Wave orbital velocities in earth-fixed *X*, *Y* and *Z* directions	m/s
$\dot{u}_w, \dot{v}_w, \dot{w}_w$	Wave orbital accelerations in earth-fixed *X*, *Y* and *Z* directions	m/s^2
Z'	Modified *Z* position for pressure stretching	m
Z_{jk}	Quadratic transfer function	
α	Wheeler's depth decay modifier	
β	Direction of wave propagation measured from the positive *X*-axis counterclockwise about the *Z*-axis	radians
β_0	Direction of primary wave system	radians
β_A	Spreading angle	radians
β_L	Lower limit on wave component direction for spread seas	radians
β_U	Upper limit on wave component direction for spread seas	radians
β_W	Direction of wind measured from the positive *X*-axis counterclockwise about the *Z*-axis. If ship is traveling in the positive *X* direction, 0 will be a tail wind.	radians
ε	Wave phase angle	radians
ϕ	Velocity potential	m^2/s
γ	JONSWAP peak enhancement factor	
λ	Wave length	m
ρ	Water density	kg/m^3
ω	Wave frequency	rad/s
ω_{llim}	Lower truncation limit of the spectral frequencies	rad/s
ω_m	Modal wave frequency	rad/s

ω_{ulim}	Upper truncation limit of the spectral frequencies	rad/s
ζ	Wave elevation	m

1.2 Equations of Motion and Coordinate Systems

Symbol	Definition	SI Units
$A_{ij\infty}$	Added mass at infinite frequency in the i^{th} mode due to unit acceleration in the j^{th} direction	kg
A_{ij}'	Added mass at wave encounter frequency	kg
$\vec{F}$	Total force vector in ship-fixed reference frame	N
$\vec{F}'$	Total force vector in ship-fixed reference frame, not including weight and infinite frequency added mass force	N
I_{xx}, I_{yy}, I_{zz}	Moments of inertia about ship-fixed origin	kg m^2
I_{xy}, I_{yz}, I_{zx}	Products of inertia about ship-fixed origin	kg m^2
$\vec{M}$	Total moment vector in ship-fixed reference frame	N m
$\vec{M}'$	Total moment vector in ship-fixed reference frame, not including moment due to ships weight and infinite frequency added mass	N m
m	Mass of the ship	kg
p, q, r	Angular velocities about the three ship-fixed axes	rad/s
$\dot{p}, \dot{q}, \dot{r}$	Angular accelerations about the three ship-fixed axes	rad/s^2
u, v, w	Velocities in the x,y,z directions (ship-fixed), respectively	m/s
$\dot{u}, \dot{v}, \dot{w}$	Accelerations in the x,y,z directions (ship-fixed).	m/s^2
W	Weight of the ship (m·g)	N
X,Y,Z	Position of the origin of the ship-fixed axes system relative to the earth-fixed axes system	m
$\dot{x}, \dot{y}, \dot{z}$	Velocity of ship origin in the earth-fixed frame	m/s
ε_1, ε_2, ε_3, ε_4	The components of the quaternion	
ϕ, θ, ψ	Euler angles: roll, pitch and yaw	rad
$\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$	Time derivatives of Euler angles	

1.3 Bilge keel force module

Symbol	Definition	SI Units
A_{ϕ}	Roll amplitude	rad
$b_{BK}(x)$	Bilge keel breadth (height)	m
C_B	Block coefficient	-
C_M	Midship section coefficient	-
L_{BK}	Bilge keel length	m
L_{pp}	Length between perpendiculars	m
OG	Vertical center of gravity w.r.t. waterline	m
p	Roll velocity	rad/s
$r_{BK}(x)$	Mean distance from G to bilge keel	m
R	Bilge radius	m
$T(x)$	Sectional draft	m
U	Ship speed	m/s
U_{10}	Ship reference speed = 10m/s	m/s
VCG	Vertical center of gravity w.r.t. keel	m
x	Longitudinal coordinate	m
δ	Angle between the line connecting CG and the bilge keel root and the horizontal through CG	rad
γ	Angle between the line connecting CG and the bilge keel root and the bilge keel plane	rad
ρ	Density of water	kg/m^3
$\sigma(x)$	Sectional area coefficient	-
ω_{ϕ}	Roll frequency	rad/s

1.4 Rudder Force Module

Symbol	Definition	SI Units
a	Aspect ratio of rudder (geometric aspect ratio)	
a_e	Effective aspect ratio	
A_R	Area of rudder planform	m^2
c_r	Chord at root of rudder	m
c_t	Chord at tip of rudder	m
x_c,y_c,z_c	Location of the center of pressure of the rudder	m
x_m,y_m,z_m	Location of the midspan point along quarter chord line	m
x_r,y_r,z_r	Location of the rudder root quarter-chord point	m
x_t,y_t,z_t	Location of the rudder tip quarter-chord point	m
$\overline{b}$	Mean rudder span. Note that this is sometimes referred to as the "semi-span" such as in Whicker and Fehlner, 1958	m
$\overline{c}$	Mean rudder chord (average of tip and root chord)	m
C_L	Lift Coefficient	
C_D	Drag Coefficient	
C_{DC}	Cross flow drag coefficient	
C_{d0}	Minimum section drag coefficient (0.0065 for NACA 0015 sect.)	
C_N	Normal force coefficient for a wing in stalled condition as described in Hoerner (1975). Recommended value 1.8 to 2.0.	
$\vec{F}_R$	Force vector for rudder force in ship-fixed reference frame	N
L	Lift from rudder	N
$\vec{M}_R$	Moment vector defining moment from rudder in ship-fixed reference frame.	N m
d	Drag on rudder	N
D	Diameter of propeller	m
T	Propeller thrust	N
p, q, r	Angular velocities about the three ship-fixed axes	rad/s
u, v,w	Velocity of the ship at the ship-fixed reference frame origin in the x,y,z directions (ship-fixed), respectively.	m/s
u_w, v_w, w_w	Wave orbital velocity at the rudder, defined in the ship-fixed	m/s

	frame	
w_F	Wake fraction coefficient	-
α	Angle of attack	rad
α_S	Stall angle	rad
β	Drift angle of flow into rudder	rad
δ	Rudder deflection angle	rad
ρ	Density of water in which rudder is operating	kg/m^3
λ	Taper ratio (c_t/c_r)	-
Λ	Sweep angle of quarter-chord line	rad
γ_R	Flow straightening factor, used to adjust the angle of attack of the rudder to account for the flow straightening influence of the hull and propeller. (default value 1.0)	
ϕ_R	Dihedral angle of rudder, which is measured as the inclination from vertical with a positive angle indicating the rudder tip is further to starboard than the rudder root.	rad

1.5 Propeller Force Module

Symbol	Definition	SI Units
A_D	Area of propeller disk	m^2
A_S	Area of submerged portion of propeller disk	m^2
D	Diameter of the propeller	m
F_{Cross}	Component of the propeller force perpendicular to the propeller shaft in the direction of the cross flow	N
F_{Norm}	Component of the propeller force perpendicular to both the propeller shaft and cross flow	N
$\vec{F}_P$	Force vector for propeller force in ship-fixed reference frame	N
J	Advance coefficient	-
K_T	Propeller thrust coefficient	-
$\vec{M}_P$	Moment vector defining moment from propeller in ship-fixed reference frame	N m

n	Rotational speed of the propeller, revolutions per second	rev/s
$\vec{n}_S$	Unit vector defining direction of propeller shaft in ship-fixed frame	-
$\vec{n}_C$	Unit vector defining direction of cross flow	-
$\vec{n}_N$	Unit vector defining the direction normal to cross flow and propeller shaft	-
p, q, r	Angular velocities about the three ship-fixed axes	rad/s
R	Propeller radius	m
RHP	Integer flag defining whether propeller is right hand turning or left hand turning (+1 for right hand turning, -1 for left hand turning)	
T	Propeller thrust (component of propeller force parallel to propeller shaft)	N
t_P	Thrust deduction coefficient	
U_A	Component of velocity vector at center of propeller disk that is parallel to the propeller shaft	
$\vec{U}_P$	Total velocity vector at the center of the propeller disk	m/s
$\vec{U}_{CROSS}$	Cross flow velocity vector at the center of the propeller disk	m/s
V_A	Advance velocity into propeller disk	m/s
u, v, w	Velocity of the ship at the ship-fixed reference frame origin in the x,y,z directions (ship-fixed), respectively	m/s
u_W, v_W, w_W	Wave orbital velocity at the center of the prop disk, defined in the ship-fixed frame	
w_F	Wake fraction coefficient	
x_P, y_P, z_P	Position of the center of the propeller disk in the ship-fixed reference frame.	
Z_W^E	Wave elevation above propeller, defined in earth-fixed frame	
α_S	Angle between the x-axis of the ship-fixed frame and the projection of the shaft axis onto the vertical plane of symmetry of the ship. A positive angle indicates that the shaft points downwards	
β_S	Angle of the propeller shaft in the horizontal plane relative to the ship x-axis, positive to port.	
γ	Flow straightening factor, used to adjust ϕ_S to account for the flow straightening influence of the hull.	
ρ	Density of the fluid	
ϕ_S	Angle between shaft axis and total velocity vector at the center of the propeller disk	

2 Theory Overview

The core ‘physics engine’ within the time-domain simulator solves the system of ordinary differential equations that describe the equations of motion (for rigid bodies) and propagates the solutions forward in time. The time-domain simulator and equations of motion are described in Section 3. At each time step the total force on the ship must be computed. In order to compute the forces on the ship, the environment in which the ship is operating must first be evaluated, which may include the wind and current in addition to the wave environment. The environment model will be able to compute the pressure and velocity field at any arbitrary point. In the “Level-0” implementation of Tempest, the environment consists only of the superposition of linear long crested waves as described in Section 4. When the ship is operating in waves it is necessary to control the rudders to keep the ship on course. Tempest uses a PID controller for the rudder which is described in Section 5. In Tempest it will be assumed that the total force on the ship can be decomposed into various force contributions, and the total forces on the hull at each time step can therefore be obtained by summing the various force contributions. In the Level-0 theory, the following force contributions will be considered:

- Hydrostatic forces
- Froude-Krylov forces
- Radiation forces
- Diffraction forces
- Hull resistance
- Bilge keel forces
- Bare hull maneuvering forces (includes skeg force)
- Rudder and fin forces
- Propulsion forces
- Weight of ship

The total force on the ship at each time step is then computed as the sum of component forces. The theory used to compute each component force is described in Section 6. The total force will be used to solve the rigid body equations of motion in ship-fixed coordinate system at each time step.

3 Equations of Motion

3.1 Summary

Tempest will use a time stepping method to solve the Euler equations of motion to predict the motion of a ship. This section presents the theory for setting up the equations of motion. The equations of motion are derived from Newton's Second Law:

$$\begin{aligned} \vec{F} &= m\vec{a} \\ \vec{M} &= [I]\vec{\alpha} \end{aligned} \tag{3.1}$$

Equation (3.1) must be applied in an inertial coordinate system. It would be possible to solve these equations directly in the inertial coordinate system, however the inertia tensor, [I], as well as many of the force components, is defined in a ship-fixed axis system and would have to be derived or transformed into the inertial system at each time step. Therefore, transformations are applied to develop the equations of motion in a ship-fixed coordinate system, resulting in the Euler equations of motion for a rigid body. The derivation of the Euler equation of motion can be found in several sources including Abkowitz (1969) and Etkin (1982).

The force and moment vectors in Equation (3.1) represent the total force and moment acting on the vessel. In this document, the term "forces" will hereafter be assumed to include both forces and moments. The total force on the ship at each time step will be computed by summing the component forces described in Section 6.

The equations will first be formulated making no assumptions concerning the symmetry of the ship or the location of the center of gravity of the ship relative to the origin. In the Level-0 theory, however, it will be assumed that the ship is symmetric and that the origin of the ship-fixed coordinate system coincides with the center of gravity. These assumptions will be used to generate a simplified set of equations for implementation in the Level-0 Tempest code. No assumptions are made that the ship-fixed axis system is aligned with the principal axes of the ship, even in Level-0. This results in additional terms containing the cross products of inertia, which are often assumed to be zero.

The weight of the ship and the forces from the infinite frequency added mass terms are included in the equations of motion presented in this section. The theories for the computation of all the other force components are described in Section 6. The equations described in this document assume that the forces acting on the ship have been transformed to the ship-fixed coordinate system.

Propagating the solution to the equations of motion is the job of a numerical integrator. This propagation is often done using a linear Euler integration scheme, (i.e. $x_{i+1} = x_i + \Delta t\dot{x}$), where the dot over the x indicates differentiation with respect to time. Though this simple scheme is adequate for non-oscillatory systems when used with a sufficiently small time step, in general, higher order integrators should be used. Of course, more accurate integration schemes tend to involve more calls to the underlying force models and equations of motion. Four different integrators were developed for Tempest: Euler, Modified Euler, 4th Order Runge-Kutta, and Adams-Bashforth. Additional integrators could easily be added as new components. The equations are formulated initially for the Level-0 code, with the intention that they can probably be extended with only minor modifications to the Level-II and Level-III theory implementations of Tempest. Some of the assumptions regarding symmetry and the location of the center of gravity may be removed in the Level-II and Level-III implementations.

3.2 Coordinate Systems for Equations of Motion

The equations of motion are set up to describe the trajectory of the ship-fixed reference system relative to an earth-fixed reference system. The earth-fixed reference frame, $O_{X_E Y_E Z_E}$, is assumed to be a right-handed coordinate system with the Z-axis positive upwards and the origin on the calm water surface. The ship-fixed reference frame, $O_{x_S y_S z_S}$, is a right handed system with the *x*-axis positive forward through the bow, *y*-axis positive to port, and the *z*-axis positive upwards. In the Level-0 implementation of Tempest, the origin of the ship-fixed system coincides with the center of gravity of the ship. Later implementations may remove this requirement, so the equations will first be developed for an arbitrary location of the origin. The $O_{x_S y_S z_S}$ axis system is fixed with the ship and moves with all the motions of the ship. The $O_{X_E Y_E Z_E}$ axis system is fixed to the earth. A third axis system, $O_{x'y'z'}$, is required to compute some of the linearized seakeeping forces. This axis system moves in the earth-fixed *X* and *Y* direction and rotates with yaw to align with the origin of the ship-fixed system. This third system's *x-y* plane remains parallel to the calm water surface, and it does not heave, pitch, or roll relative to the earth-fixed system. Its origin is the same as the ship-fixed origin, though in some special cases may be directly below the ship-fixed origin on the calm water surface ($Z_E = 0$).

The solution to the equations of motion will provide the position of the ship-fixed reference frame relative to the earth-fixed frame at each time step. This can be described with six variables: the (X,Y,Z) position of the origin of the ship-fixed reference frame relative to the earth-fixed frame and the three Euler angles: ϕ, θ and ψ, which represent the roll, pitch, and yaw of the ship-fixed reference frame. The order of the rotations are important and must be performed in the order: ψ, θ, ϕ. When the rotations are taken in this order, the definitions of yaw (ψ), pitch (θ), and roll (ϕ) are as follows:

- yaw (ψ): The angle between the vertical plane defined by the earth-fixed *X*, *Z* plane and the earth-fixed vertical plane passing through the ship-fixed *x* axis (the plane defined by the ship-fixed *x* axis and the earth-fixed *Z* axis translated to the ship-fixed origin). Positive yaw is defined using a right hand rule about the *Z* axis.
- pitch (θ): The angle of elevation of the ship-fixed *x* axis relative to the earth-fixed *X*, *Y* plane. Positive pitch is bow down.
- roll (ϕ): The angle between the ship-fixed *x*,*z* plane and the earth-fixed vertical plane passing through the ship-fixed *x* axis (the plane defined by the ship-fixed *x* axis and the earth-fixed *Z* axis translated to the ship-fixed origin). Positive roll angle is defined using a right hand rule about the *x* axis. Positive roll is to starboard.

When transforming translational velocities, forces and moments from the ship-fixed frame to the earth-fixed frame, the transformation matrix $[T_{S/E}]$ is used, where this matrix is defined as:

$$[T_{S/E}] = \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi + \sin\theta\sin\phi\cos\psi & \sin\phi\sin\psi + \sin\theta\cos\phi\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & -\sin\phi\cos\psi + \sin\theta\cos\phi\sin\psi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \quad (3.2)$$

Alternatively, the quaternion representation can be used to formulate the position and orientation of the ship-fixed frame relative to the earth-fixed frame can be formulated in terms of the (*X*,*Y*,*Z*) position of the origin of the ship-fixed reference frame relative to the earth-fixed frame and the quaternion, ε. The quaternion is formed by four components:

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix} \tag{3.3}$$

The transformation matrix $[T_{S/E}]$ can be formulated in terms of the quaternion as:

$$\left[T_{S/E}\right] = \begin{bmatrix} 1-2(\varepsilon_2^2+\varepsilon_3^2) & 2(\varepsilon_1\varepsilon_2-\varepsilon_3\varepsilon_4) & 2(\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_2+\varepsilon_3\varepsilon_4) & 1-2(\varepsilon_1^2+\varepsilon_3^2) & 2(\varepsilon_2\varepsilon_3-\varepsilon_1\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4) & 2(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4) & 1-2(\varepsilon_1^2+\varepsilon_2^2) \end{bmatrix} \tag{3.4}$$

The quaternion components satisfy the constraint equation:

$$\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2+\varepsilon_4^2 = 1 \tag{3.5}$$

The quaternion components can be computed from the Euler angles and vice versa. The equations to obtain the quaternion from the Euler angles are:

$$\begin{aligned}
\varepsilon_1 &= \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) - \sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) \\
\varepsilon_2 &= \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) + \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) \\
\varepsilon_3 &= \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) - \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) \\
\varepsilon_4 &= \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) + \sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)
\end{aligned} \tag{3.6}$$

For most cases the initial roll, pitch and yaw will be zero, in which case the initial value of the quaternion is (0,0,0,1). The equations used to obtain the Euler angles from the quaternion are:

$$\begin{aligned}
\phi &= \arctan\left(\frac{2(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4)}{1-2(\varepsilon_1^2+\varepsilon_2^2)}\right) \\
\theta &= \arcsin\left(2(\varepsilon_2\varepsilon_4-\varepsilon_1\varepsilon_3)\right) \\
\psi &= \arctan\left(\frac{2(\varepsilon_1\varepsilon_2+\varepsilon_3\varepsilon_4)}{1-2(\varepsilon_2^2+\varepsilon_3^2)}\right)
\end{aligned} \tag{3.7}$$

The transformation matrix [$T_{S/E}$] is used to transform the position or translational velocity of point in the ship-fixed frame to the earth-fixed frame. To compute the earth-fixed translational velocities the formula is:

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \left[T_{S/E}\right] \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{3.8}$$

To convert the x,y,z position of a point defined in the ship-fixed frame to the earth-fixed frame, the formula is:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{Earth-fixed}} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \left[T_{S/E}\right] \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{Ship-fixed}} \tag{3.9}$$

where the (X,Y,Z) vector on the left hand side is the position of the point in the earth-fixed frame, the first (X,Y,Z) vector on the right hand side is the position of the origin of the ship-fixed frame defined in the earth-fixed frame, and the last (x,y,z) vector is the position of the point in the ship-fixed frame. The matrix [$T_{S/E}$] is orthogonal, so its inverse is simply the transpose of the matrix, which can be used to perform transformations in the opposite direction to convert quantities from the earth-fixed frame to the ship-fixed frame.

$$\begin{aligned} \left[T_{E/S}\right] = \left[T_{S/E}\right]^T &= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\theta\sin\phi\cos\psi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\theta\sin\phi \\ \sin\phi\sin\psi + \sin\theta\cos\phi\cos\psi & -\sin\phi\cos\psi + \sin\theta\cos\phi\sin\psi & \cos\theta\cos\phi \end{bmatrix} \\ &= \begin{bmatrix} 1-2(\varepsilon_2^2+\varepsilon_3^3) & 2(\varepsilon_1\varepsilon_2+\varepsilon_3\varepsilon_4) & 2(\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_2-\varepsilon_3\varepsilon_4) & 1-2(\varepsilon_1^2+\varepsilon_3^3) & 2(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_4) & 2(\varepsilon_2\varepsilon_3-\varepsilon_1\varepsilon_4) & 1-2(\varepsilon_1^2+\varepsilon_2^3) \end{bmatrix} \end{aligned} \tag{3.10}$$

and:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \left[T_{E/S}\right] \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} \tag{3.11}$$

A different set of equations are required to compute the time derivatives of the Euler angles from the angular velocities defined in the ship-fixed axis system and vise-versa. Note that the derivatives of the Euler angles are not the rotational velocities with respect to the earth-fixed reference frame, but rather the rate of change in the roll, pitch and yaw as defined above. The Euler angle derivatives do not form an angular velocity vector, as they do not represent rotations about an orthogonal set of axes. Therefore the matrix that is used to transform is not orthogonal. The following transformation matrix is used to compute the rotational velocity vector in the ship-fixed frame, (p,q,r), when the Euler angle derivatives $(\dot{\phi},\dot{\theta},\dot{\psi})$, are known:

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{pmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{pmatrix} \tag{3.12}$$

The matrix can be inverted to compute the Euler angle derivatives $(\dot\phi, \dot\theta, \dot\psi)$ from the rotational velocity vector in the ship-fixed frame (p,q,r):

$$\begin{pmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{pmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{3.13}$$

There is a singularity in the transformation matrix shown above when the pitch angle of the ship is ±90°, however this is not expected to be a problem for a surface ship, even if the ship is operating in very large waves. This problem can be avoided when the quaternion formulation is used. With the quaternion representation, the time derivative of the quaternion is computed instead of the time derivative of the Euler angles. The time derivate of the Euler angles can be computed from (p,q,r):

$$\begin{pmatrix} \dot\varepsilon_1 \\ \dot\varepsilon_2 \\ \dot\varepsilon_3 \\ \dot\varepsilon_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & r & -q & p \\ -r & 0 & p & q \\ q & -p & 0 & r \\ -p & -q & -r & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix} \tag{3.14}$$

Finally a transformation matrix may be required to transform forces and velocities from the yawed upright axis system, $O_{x'y'z'}$, to the ship-fixed axis system. The matrix used to transform quantities from the yawed upright frame to the ship-fixed frame is the matrix [$T_{U/S}$] which is shown below. This matrix is orthogonal, so its inverse is simply the transpose of the matrix.

$$[T_{U/S}] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{3.15}$$

$$[T_{S/U}] = [T_{U/S}]^T = \begin{bmatrix} \cos\theta & \sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi & -\sin\phi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \tag{3.16}$$

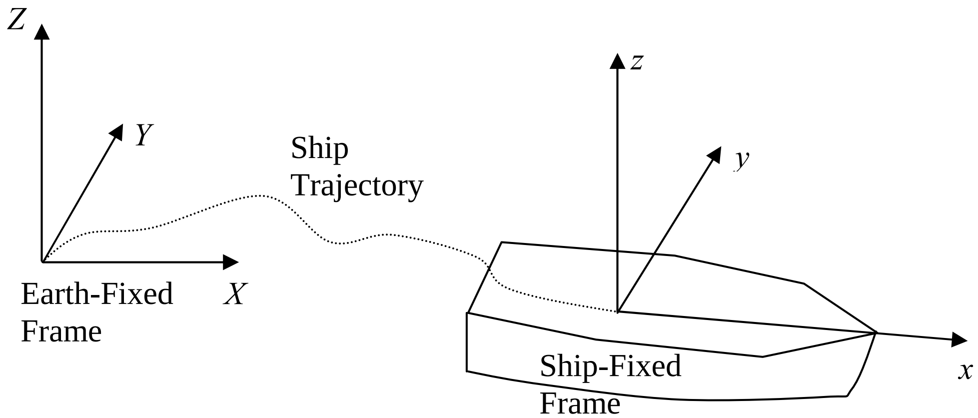

Figure 3-1 Definition of reference frames used in equations of motion

3.2.1 Heading Definitions

Tempest allows the ship to head in any direction within the earth-fixed coordinate system, and likewise allows waves to travel in any direction. The ship heading is defined as the angle ψ, which is equivalent to the yaw angle defined in the Section 3.2. The heading of a wave system in earth-fixed coordinates is defined by the angle β. The relative wave heading, which is the heading of the wave system relative to the ship's heading, is given by:

$$\mu_n = \beta_n - \psi \tag{3.17}$$

The ship heading, wave heading, and relative wave heading, are shown in Figure 3-2.

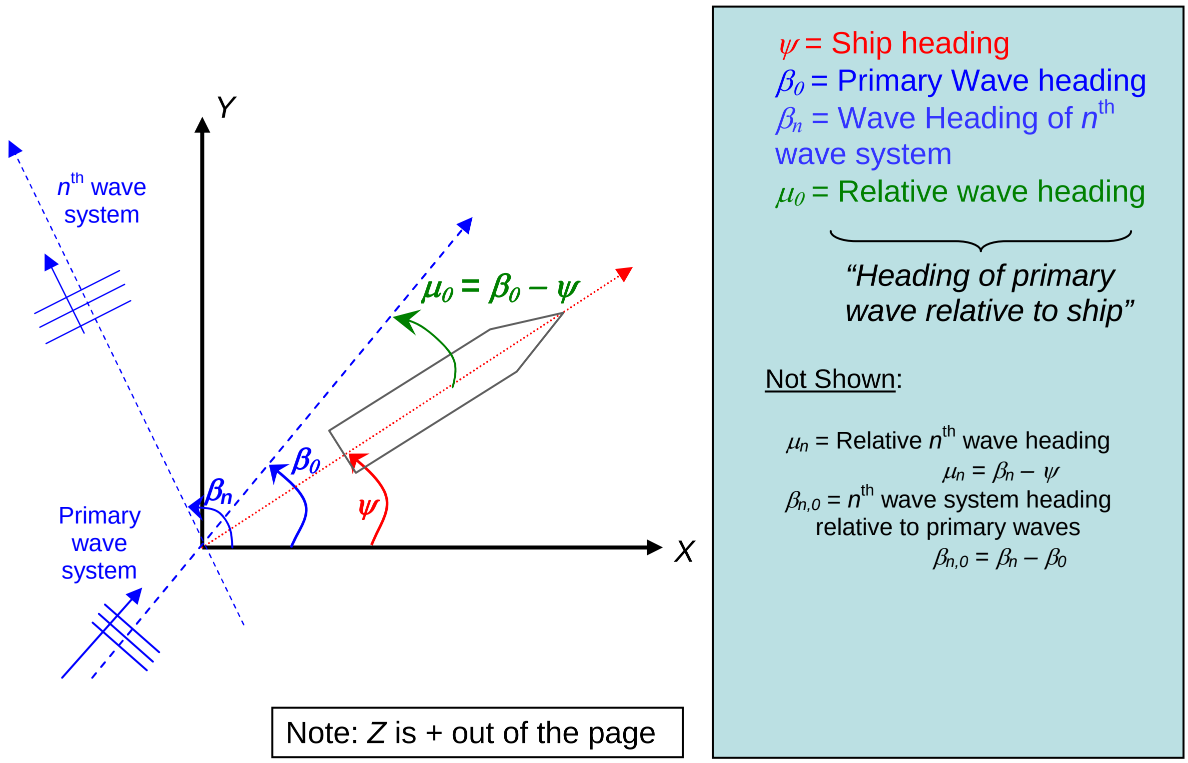

Figure 3-2 Definition of ship heading and wave heading angles

3.3 General Formulation

In Tempest the equations of motion using both an Euler angle formulation and a quaternion formulation will be implemented. The majority of the implantation is identical for both formulations, with the primary difference being that the time integration is performed on the derivatives of the Euler angles or quaternion components. The equations of motion for a six-degree-of-freedom rigid body can be derived from Newton's Laws of Motion (see Abkowitz (1969) and Etkin (1982), for example). If the mass of the body and the mass distribution are constant over time, then the equations can be written as:

$$m\left(\dot{u} + qw - rv - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right) = F_x + W\sin\theta$$

$$m\left(\dot{v} + ru - pw - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right) = F_y - W\cos\theta\sin\phi$$

$$m\left(\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})\right) = F_z - W\cos\theta\cos\phi$$

$$I_{xx}\,\dot{p} + \left(I_{zz} - I_{yy}\right)qr + \left(pr - \dot{q}\right)I_{xy} - \left(\dot{r} + pq\right)I_{zx} + \left(r^2 - q^2\right)I_{yz} + m\left[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)\right]$$
$$= M_x - y_G W\cos\theta\cos\phi + z_G W\cos\theta\sin\phi$$

$$I_{yy}\,\dot{q} + \left(I_{xx} - I_{zz}\right)rp - \left(\dot{p} + qr\right)I_{xy} + \left(p^2 - r^2\right)I_{zx} + \left(qp - \dot{r}\right)I_{yz} + m\left[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)\right]$$
$$= M_y + x_G W\cos\theta\cos\phi + z_G W\sin\theta$$

$$I_{zz}\ \dot{r} + \left(I_{yy} - I_{xx}\right) pq + \left(q^2 - p^2\right) I_{xy} + \left(rq - \dot{p}\ \right) I_{zx} - \left(\dot{q} + rp\right)\ I_{yz} + m\left[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)\right]$$
$$= M_z - x_G W \cos\theta \sin\phi - y_G W \sin\theta$$

(3.18)

The Euler angle terms that appear in the above equations result from the transformation of the weight force vector from the earth-fixed frame to the ship-fixed frame. If the quaternion formulation is used, the relations shown below can be substituted to formulate the equations in terms of the quaternion components.

$$\begin{aligned} \sin\theta &= 2(\varepsilon_2\varepsilon_4 - \varepsilon_1\varepsilon_3) \\ \cos\theta \sin\phi &= 2(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4) \\ \cos\theta\cos\phi &= 1 - 2(\varepsilon_1^2 + \varepsilon_2^3) \end{aligned} \tag{3.19}$$

It has been found that moving the terms related to the infinite frequency added mass to the left hand side of the equations of motion improves the stability of the system. This will be implemented in Tempest. In the Level-0 implementation it will be the total added mass, obtained by interpolating the added mass at the wave encounter frequency, that will be included on the left hand side of the equations of motion, instead of the infinite frequency added mass terms. In almost all cases, the geometry of the ship will be symmetrical, in which case many of the added mass terms in the above matrix will be zero. Any added mass coefficient with one even subscript and one odd subscript will be zero for an upright symmetric ship on the calm water surface. For the "Level-0" theory, the added mass and damping coefficients will be computed using a method that is linearized about this condition, and in this case A_{12}, A_{14}, A_{24}, etc can be assumed to be zero. However, in higher levels of theory this linear approximation may not be used, and symmetry cannot be used when computing the added mass and damping terms for a heeled ship. For this reason, all the terms in the added mass and damping matrices will be included in the general derivation. A simplified derivation for implementation in "Level-0" will be provided in the following section.

The six equations will be formulated to solve for the acceleration terms in each mode, with terms related to the velocity and displacement in each mode appearing on the right hand side, computed using values from previous or intermediate time steps depending on the time integration scheme used. In matrix form the equations will appear as shown below:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} rhs_1 \\ rhs_2 \\ rhs_3 \\ rhs_4 \\ rhs_5 \\ rhs_6 \end{bmatrix} \tag{3.20}$$

The coefficients of the left hand side matrix are defined below.

$$a_{11} = m + A_{11\infty}$$
$$a_{12} = A_{12\infty}$$
$$a_{13} = A_{13\infty}$$
$$a_{14} = A_{14\infty}$$
$$a_{15} = z_G m + A_{15\infty}$$
$$a_{16} = - y_G m + A_{16\infty}$$

$$a_{21} = A_{21\infty}$$
$$a_{22} = m + A_{22\infty}$$
$$a_{23} = A_{23\infty}$$
$$a_{24} = -z_G m + A_{24\infty}$$
$$a_{25} = A_{25\infty}$$
$$a_{26} = x_G m + A_{26\infty}$$

$$a_{31} = A_{21\infty}$$
$$a_{32} = A_{32\infty}$$
$$a_{33} = m + A_{33\infty}$$
$$a_{34} = y_G m + A_{34\infty}$$
$$a_{35} = -x_G m + A_{35\infty}$$
$$a_{36} = A_{36\infty}$$

$$a_{41} = A_{41\infty}$$
$$a_{42} = -z_G m + A_{42\infty}$$
$$a_{43} = y_G m + A_{43\infty}$$
$$a_{44} = I_{44} + A_{44\infty}$$
$$a_{45} = -I_{xy} + A_{45\infty}$$
$$a_{46} = -I_{zx} + A_{46\infty}$$

$$a_{51} = Z_G m + A_{51\infty}$$
$$a_{52} = A_{52\infty}$$
$$a_{53} = -x_G m + A_{53\infty}$$
$$a_{54} = -I_{xy} + A_{54\infty}$$
$$a_{55} = I_{yy} + A_{55\infty}$$
$$a_{56} = -I_{zx} + A_{56\infty}$$

$$a_{61} = -y_G m + A_{61\infty}$$
$$a_{62} = x_G m + A_{62\infty}$$
$$a_{63} = A_{63\infty}$$
$$a_{64} = -I_{zx} + A_{64\infty}$$
$$a_{65} = -I_{yz} + A_{65\infty}$$
$$a_{66} = I_{zz} + A_{66\infty}$$

$$
\begin{aligned}
rhs_1 &= F'_x + W\sin\theta - m\left[qw - rv - x_G(q^2 + r^2) + y_G pq + z_G pr\right] \\
rhs_2 &= F'_y - W\cos\theta\sin\phi - m\left[ru - pw - y_G(r^2 + p^2) + z_G qr + x_G qp\right] \\
rhs_3 &= F'_z - W\cos\theta\sin\theta - m\left[pv - qu - z_G(p^2 + q^2) + x_G rp + y_G rq\right] \\
rhs_4 &= M'_x - y_G W\cos\theta\cos\phi + z_G W\cos\theta\sin\phi - \left(I_{zz} - I_{yy}\right)qr - pr\, I_{xy} - pq\, I_{zx} - \left(r^2 - q^2\right) I_{yz} \\
&\quad - m\left[y_G(uq + vp) - z_G(wp + ur)\right] \\
rhs_5 &= M'_y + x_G W\cos\theta\cos\phi + z_G W\sin\theta - \left(I_{xx} - I_{zz}\right)rp - qr\, I_{xy} - \left(p^2 - r^2\right)I_{zx} - qp\, I_{yz} \\
&\quad - m\left[z_G(vr + wq) - x_G(uq + vp)\right] \\
rhs_6 &= M'_z - x_G W\cos\theta\sin\phi - y_G W\sin\theta - \left(I_{yy} - I_{xx}\right)pq - \left(q^2 - p^2\right)I_{xy} - rq\, I_{zx} + (rp)\, I_{yz} \\
&\quad - m\left[x_G(wp + ur) - y_G(vr + wq)\right]
\end{aligned}
\tag{3.21}
$$

F' and M' are the total forces and moments not including the weight of the ship and the force contribution from the infinite frequency added mass terms.

At every time step, the six equations listed above in matrix form are solved to compute the accelerations in the ship-fixed frame, $(\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r})$. The ship-fixed accelerations are integrated with time to obtain the ship-fixed velocities (u,v,w,p,q,r).

$$
\begin{aligned}
u &= u_0 + \int \dot{u}\,dt \quad , \quad v = v_0 + \int \dot{v}\,dt \quad , \quad w = w_0 + \int \dot{w}\,dt \\
p &= p_0 + \int \dot{p}\,dt \quad , \quad q = q_0 + \int \dot{q}\,dt \quad , \quad r = r_0 + \int \dot{r}\,dt
\end{aligned}
\tag{3.22}
$$

Here $(u_0,v_0,w_0,p_0,r_0,q_0)$ are the initial translational and rotational velocities in the ship-fixed frame.

Up to this point the procedure is identical for the Euler angle formulation and the quaternion formulation. The next step is where the two formulations differ. In the Euler angle formulation, the time derivatives of the Euler angles derivatives $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ are computed from the ship-fixed rotation velocities (p,q,r), using Eqn. (3.13). The time integration is performed on the Euler angle derivatives to obtain the Euler angles at the current time step:

$$
\phi = \phi_0 + \int \dot{\phi}\,dt \quad , \quad \theta = \theta_0 + \int \dot{\theta}\,dt \quad , \quad \psi = \psi_0 + \int \dot{\psi}\,dt
\tag{3.23}
$$

where ϕ_0, θ_0, and ψ_0 are the initial roll, pitch and yaw angles of the ship.

In the quaternion formulation, the time derivatives of the quaternion components are computed instead of the time derivatives of the Euler angles, and then the time integration is performed for the quaternion instead of the Euler angles.

$$\varepsilon_1 = \varepsilon_{1\_0} + \int \dot{\varepsilon}_1\, dt \;\;,\;\; \varepsilon_2 = \varepsilon_{2\_0} + \int \dot{\varepsilon}_2\, dt$$
$$\varepsilon_3 = \varepsilon_{3\_0} + \int \dot{\varepsilon}_3\, dt \;\;,\;\; \varepsilon_4 = \varepsilon_{4\_0} + \int \dot{\varepsilon}_4\, dt \qquad (3.24)$$

The _0 subscript indicates the initial value. For the quaternion formulation, numerical integration errors accumulate so that the constraint imposed by in Eqn. (3.5) is no longer satisfied. The quaternion should be re-normalized at regular intervals during the simulation to satisfy Eqn. (3.5).

The transformation matrix $[T_{S/E}]$ is then computed using either Eqn. (3.2) in the Euler angle formulation or Eqn. (3.4) in the quaternion formulation. The ship-fixed translational velocities (u,v,w) are then transformed to the translational velocities in the earth-fixed frame $(\dot{x}, \dot{y}, \dot{z})$, and then integrated to obtain the position of the ship relative to the earth-fixed frame.

$$X = X_0 + \int \dot{X}\, dt \quad , \quad Y = Y_0 + \int \dot{Y}\, dt \quad , \quad Z = Z_0 + \int \dot{Z}\, dt \qquad (3.25)$$

where (X_0,Y_0,Z_0) is the initial position of the ship-fixed origin defined in the earth-fixed frame.

3.4 Restricting degrees of freedom

In certain cases it may be desired to restrict the ship in one or more degrees of freedom. This may be necessary to simulate a model test where the model was free to move only in some degrees of freedom, or to compute forces on a ship with a prescribed motion. The restriction on the motion of the ship can be applied either in the earth-fixed frame or in the ship-fixed frame. In the case where the restrictions are imposed in the ship-fixed frame, the time history of the ship-fixed acceleration in the restricted degrees of freedom will be provided as input. Alternatively the time history of the ship-fixed velocity can be specified as input and the accelerations computed by differentiation. Eqn. (3.20) will still be used to solve for all six accelerations in the ship-fixed frame. However, the ship-fixed acceleration component corresponding to restricted degrees of freedoms $\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}\ or\ \dot{r}$ will be set to the user-specified value prior to performing the integration to compute the ship-fixed velocities. The computed values for ship-fixed accelerations from solving Eqn. (3.20) can be used to compute the forces and moments that must be applied to restrict the ship in the specified degree of freedoms.

In some instances, such as simulating a model test where the ship is restricted by using a heave staff, it is the motion in the earth-fixed frame that is restricted. In this case one or more components of translational velocity vector in the earth-fixed frame, $(\dot{X}, \dot{Y}, \dot{Z})$, or one or more of the time derivatives of the Euler angles, $(\dot{\phi}, \dot{\theta}, \dot{\psi})$, must be provided as input. For this case, the ship-fixed accelerations and velocities will first be computed without any restrictions. Then the unrestricted values for $(\dot{X}, \dot{Y}, \dot{Z})$ are computed using Eqn. (3.8) and the unrestricted values for $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ are computed using Eqn. (3.13). The values for $(\dot{X}, \dot{Y}, \dot{Z})$ and $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ in the restricted modes are then replaced with their prescribed values. The ship-fixed velocities, (u,v,w,p,q,r), including the effects of the restricted motion can then be computed using Eqns. (3.11) and (3.12).

3.5 Simplified Formulation for Level-0 Implementation

In the Level-0 implementation of Tempest, several assumptions will be made which result in a simpler form of the equation of motion. These assumptions are:

- The origin of the ship-fixed frame is at the center of gravity of the ship
- The geometry of the ship is symmetric, so any added mass terms with both an even and odd subscript can be set to zero.
- The complete added mass interpolated at the encounter frequency will be included on the left hand side of the equations in Level-0, as described in the Section 6.2.1.3.

With these assumptions, the equations of motion simplify to:

$$m\left(\dot{u}+qw-rv\right)=F_x+W\sin\theta$$

$$m\left(\dot{v}+ru-pw\right)=F_y-W\cos\theta\sin\phi$$

$$m\left(\dot{w}+pv-qu\right)=F_z-W\cos\theta\cos\phi$$

$$I_{xx}\ \dot{p}+\left(I_{zz}-I_{yy}\right)qr+\left(pr-\dot{q}\right)I_{xy}-\left(\dot{r}+pq\right)I_{zx}+\left(r^2-q^2\right)I_{yz}=M_x$$

$$I_{yy}\ \dot{q}+\left(I_{xx}-I_{zz}\right)rp-\left(\dot{p}+qr\right)I_{xy}+\left(p^2-r^2\right)I_{zx}+\left(qp-\dot{r}\right)I_{yz}=M_y$$

$$I_{zz}\ \dot{r}+\left(I_{yy}-I_{xx}\right)pq+\left(q^2-p^2\right)I_{xy}+\left(rq-\dot{p}\right)I_{zx}-\left(\dot{q}+rp\right)I_{yz}=M_z \qquad (3.26)$$

Adding the added mass terms described in Section 6.2.1.3 to the left hand side and rewriting in matrix form, these equations can be written as:

$$\left[\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{xy} & I_{yy} & -I_{yz} \\ 0 & 0 & 0 & -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}+\begin{bmatrix} A_{11}' & A_{12}' & A_{13}' & A_{14}' & A_{15}' & A_{16}' \\ A_{21}' & A_{22}' & A_{23}' & A_{24}' & A_{25}' & A_{26}' \\ A_{31}' & A_{32}' & A_{33}' & A_{34}' & A_{35}' & A_{36}' \\ A_{41}' & A_{42}' & A_{43}' & A_{44}' & A_{45}' & A_{46}' \\ A_{51}' & A_{52}' & A_{53}' & A_{54}' & A_{55}' & A_{56}' \\ A_{61}' & A_{62}' & A_{63}' & A_{64}' & A_{65}' & A_{66}' \end{bmatrix}\right]\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix}=\begin{pmatrix} rhs_1 \\ rhs_2 \\ rhs_3 \\ rhs_4 \\ rhs_5 \\ rhs_6 \end{pmatrix} \qquad (3.27)$$

where,

$$rhs_1=F_x'+W\sin\theta-m[qw-rv]$$
$$rhs_2=F_y'-W\cos\theta\sin\phi-m[ru-pw]$$
$$rhs_3=F_z'-W\cos\theta\sin\theta-m[pv-qu]$$
$$rhs_4=M_x'-\left(I_{zz}-I_{yy}\right)qr-prI_{xy}-pqI_{zx}-\left(r^2-q^2\right)I_{yz}\Big)$$
$$rhs_5=M_y'-\left(I_{xx}-I_{zz}\right)rp-qrI_{xy}-\left(p^2-r^2\right)I_{zx}-qpI_{yz}\Big)$$
$$rhs_6=M_z'-\left(I_{yy}-I_{xx}\right)pq-\left(q^2-p^2\right)I_{xy}-rqI_{zx}-rpI_{yz}$$

F' and M' are now the total forces and moments not including the weight of the ship and the force contribution from the added mass terms. F' and M' are computed by summing the component forces described in Section 6 according to Equation (6.2). The total added mass force contribution is now included on the left hand side by the A_{ij}' terms. After computing the accelerations in the ship-fixed frame $(\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r})$, the procedure for integrating the accelerations to compute the velocities and integrating the velocities to compute the ship trajectory is identical to that listed in the previous section.

4 Linear Wave Environment for Tempest

4.1 Introduction

This section details the theory for computing the wave elevation, pressures, and kinematics for a linear seaway to be used in the Tempest dynamic stability and seakeeping simulation code. The kinematics for a linear regular (sine) wave travelling in an arbitrary direction in infinite and finite depth water are presented. The definition for an irregular sea is built upon these kinematics and includes both long-crested (unidirectional) and short-crested (directional spread) seaways. The formulations for several theoretical spectra are included. Specifications for input and output parameters follow the development of the theory.

4.2 Coordinate System

The coordinate system used to describe the wave field is an earth-fixed coordinate system defined by a right hand rule with Z positive up. Z is equal to zero on the calm water line. A wave direction of $\beta = 0$ means that a wave progresses along the X-axis.

4.3 Relationships for Infinite Depth Linear Regular Waves

Wave Number:

$$k = \frac{2 \cdot \pi}{\lambda} = \frac{\omega^2}{g} \tag{4.1}$$

Wave Length:

$$\lambda = \frac{2 \cdot \pi}{k} = \frac{2 \cdot \pi \cdot g}{\omega^2} = \frac{g \cdot T^2}{2 \cdot \pi} \tag{4.2}$$

Frequency:

$$\omega = 2 \cdot \pi \cdot f = \sqrt{g \cdot k} = \frac{2 \cdot \pi}{T} = \sqrt{\frac{2 \cdot \pi \cdot g}{\lambda}} \tag{4.3}$$

Wave Celerity (Phase Velocity):

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}} = \frac{g}{\omega} = \frac{g}{2 \cdot \pi \cdot f} \tag{4.4}$$

Group Velocity:

$$c_g = \frac{1}{2} \cdot c \tag{4.5}$$

4.4 Relationships for Finite Depth Linear Regular Waves

Wave Number:

$$k = \frac{2 \cdot \pi}{\lambda} \qquad \frac{\omega^2}{g} = k \cdot \tanh(k \cdot h) \tag{4.6}$$

Wave Length:

$$\lambda = \frac{2 \cdot \pi}{k} = \frac{2 \cdot \pi \cdot g}{\omega^2} \cdot \tanh\left(\frac{2 \cdot \pi \cdot h}{\lambda}\right) = \frac{g}{2 \cdot \pi} \cdot T^2 \cdot \tanh\left(\frac{2 \cdot \pi \cdot h}{\lambda}\right) \tag{4.7}$$

Frequency:

$$\omega = 2 \cdot \pi \cdot f = \frac{2 \cdot \pi}{T} = \sqrt{g \cdot k \cdot \tanh(k \cdot h)} = \sqrt{\left(\frac{2 \cdot \pi \cdot g}{\lambda} \cdot \tanh\left(\frac{2 \cdot \pi \cdot h}{\lambda}\right)\right)} \tag{4.8}$$

Wave Celerity (Phase Velocity):

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k} \cdot \tanh(k \cdot h)} = \frac{g}{\omega} \cdot \tanh(k \cdot h) = \frac{g}{2 \cdot \pi \cdot f} \tag{4.9}$$

Group Velocity:

$$\begin{gathered} c_g = \frac{\omega}{k} \cdot \left(\frac{1}{2} + \frac{k \cdot h}{\sinh(2 \cdot k \cdot h)}\right) \\ c_g \to c \text{ as } h \to 0 \\ c_g \to \frac{1}{2} c \text{ as } h \to \infty \end{gathered} \tag{4.10}$$

4.5 Regular-Linear-Wave Velocity Potential and Kinematics

The 3D velocity potential of a regular wave traveling in an arbitrary direction in finite and infinite depth is presented in the following sections. One can easily reduce these equations to a 2D wave by setting the wave direction, β, to 0.

4.6 3D Velocity Potential and Kinematics for a Single Wave – Infinite-Depth

Velocity Potential:

$$\varphi = \frac{i \cdot g \cdot a}{\omega} \cdot e^{k \cdot Z} \cdot e^{i \cdot (\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon)} \tag{4.11}$$

Real Part of the Velocity Potential:

$$\Re(\varphi) = \frac{-g \cdot a}{\omega} \cdot e^{k \cdot Z} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \tag{4.12}$$

Wave Elevation:

$$\zeta = \frac{-1}{g} \cdot \left(\frac{\partial}{\partial t} \Re(\varphi) \right) = a \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \tag{4.13}$$

Hydrostatic Pressure:

$$p_{hys} = -\rho \cdot g \cdot Z \tag{4.14}$$

Dynamic Pressure:

$$p_D = -\rho \cdot \left(\frac{\partial}{\partial t} \Re(\varphi) \right) = \rho \cdot g \cdot a \cdot e^{k \cdot Z} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \tag{4.15}$$

Particle Orbital Velocity Components:

$$\begin{aligned} \vec{V} &= \{u_w, v_w, w_w\} = \nabla \Re(\varphi) = \left(\frac{\partial}{\partial X} \Re(\varphi), \frac{\partial}{\partial Y} \Re(\varphi), \frac{\partial}{\partial Z} \Re(\varphi) \right) \\ u_w &= a \cdot \omega \cdot e^{k \cdot Z} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \cos(\beta) \\ v_w &= a \cdot \omega \cdot e^{k \cdot Z} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \sin(\beta) \\ w_w &= -a \cdot \omega \cdot e^{k \cdot Z} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \end{aligned} \tag{4.16}$$

Particle Acceleration Components:

$$\begin{aligned} \dot{\vec{V}} &= \{\dot{u}_w, \dot{v}_w, \dot{w}_w\} = \frac{\partial \vec{V}}{\partial t} = \frac{\partial}{\partial t} \left(\nabla \Re(\varphi) \right) \\ \dot{u}_w &= -a \cdot \omega^2 \cdot e^{k \cdot Z} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \cos(\beta) \\ \dot{v}_w &= a \cdot \omega^2 \cdot e^{k \cdot Z} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \sin(\beta) \\ \dot{w}_w &= -a \cdot \omega^2 \cdot e^{k \cdot Z} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \end{aligned} \tag{4.17}$$

4.7 3D Velocity Potential and Kinematics for a Single Wave – Finite Depth

Velocity Potential:

$$\varphi = \frac{i \cdot g \cdot a}{\omega} \cdot \frac{\cosh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot e^{i \cdot (\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon)} \tag{4.18}$$

Real Part of the Velocity Potential:

$$\Re(\varphi) = \frac{-g \cdot a}{\omega} \cdot \frac{\cosh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \tag{4.19}$$

Wave Elevation:

$$\zeta = \frac{-1}{g} \cdot \left(\frac{\partial}{\partial t} \Re(\varphi) \right) = a \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \tag{4.20}$$

Hydrostatic Pressure:

$$p_{hys} = -\rho \cdot g \cdot Z \tag{4.21}$$

Dynamic Pressure:

$$p_D = -\rho \cdot \left(\frac{\partial}{\partial t} \Re(\varphi) \right) = \rho \cdot g \cdot a \cdot \frac{\cosh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \tag{4.22}$$

Particle Orbital Velocity Components:

$$\vec{V} = \{u_w, v_w, w_w\} = \nabla \Re(\varphi) = \left(\frac{\partial}{\partial X} \Re(\varphi), \frac{\partial}{\partial Y} \Re(\varphi), \frac{\partial}{\partial Z} \Re(\varphi) \right)$$

$$\begin{aligned}
u_w &= \frac{a \cdot g \cdot k}{\omega} \cdot \frac{\cosh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \cos(\beta) \\
v_w &= \frac{a \cdot g \cdot k}{\omega} \cdot \frac{\cosh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \sin(\beta) \\
w_w &= \frac{-a \cdot g \cdot k}{\omega} \cdot \frac{\sinh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon)
\end{aligned} \tag{4.23}$$

Particle Acceleration Components:

$$\vec{\dot{V}} = \{\dot{u}_w, \dot{v}_w, \dot{w}_w\} = \frac{\partial \vec{V}}{\partial t} = \frac{\partial}{\partial t} \left(\nabla \Re(\varphi) \right)$$

$$\begin{aligned}
\dot{u}_w &= -a \cdot g \cdot k \cdot \frac{\cosh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \cos(\beta) \\
\dot{v}_w &= -a \cdot g \cdot k \cdot \frac{\cosh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \sin(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon) \cdot \sin(\beta) \\
\dot{w}_w &= -a \cdot g \cdot k \cdot \frac{\sinh(k \cdot (h + Z))}{\cosh(k \cdot h)} \cdot \cos(\omega \cdot t - k \cdot X \cdot \cos(\beta) - k \cdot Y \cdot \sin(\beta) + \varepsilon)
\end{aligned} \tag{4.24}$$

Note that the finite-depth dispersion relation is different than the infinite-depth case and one cannot substitute $k = \omega^2/g$ when deriving/defining the kinematics for the finite-depth case.

4.8 Pressure Stretching

Linear wave theory has a deficiency in that the total pressure (dynamic plus hydrostatic) does not approach zero as the free surface is approached from below. There are several methods to deal with this issue that modify the dynamic pressure, orbital velocity, and acceleration components.

Wheeler's Method for stretching can be written as a modification to Z in the dynamic pressure and orbital velocity equations. The value Z' is computed and used in place of Z to compute the dynamic pressure, particle velocities, and particle accelerations. The hydrostatic pressure is not stretched and uses Z, not Z'.

$$
\begin{aligned}
&Z' = \alpha(Z-\zeta) \\
&\alpha = \begin{cases} 1 & \text{for infinite depth} \\ \dfrac{h}{h+\zeta} & \text{for finite depth} \end{cases}
\end{aligned}
\tag{4.25}
$$

In the case of irregular waves, ζ in the above equation is the total free surface elevation comprised of all wave components from all seaways present in the simulation.

4.9 Irregular Wave Kinematics – Linear Model

Irregular waves are computed by assuming a power spectrum for a seaway. From the spectrum, individual wave components are generated. The characteristics of these wave components are summed to compute the total wave characteristics.

4.10 Long-Crested Infinite Depth Irregular Seaway Kinematics

An irregular seaway is modeled as a superposition of regular, sinusoidal waves, as defined in previous sections. The amplitudes of the wave components that compose the irregular seaway are derived from a power spectrum, $S(\omega)$, which can be derived from measurements or theoretical spectra (which are defined in a subsequent section).

The amplitudes for the wave components are derived from a discretized power spectrum and are computed as follows:

$$a_i = \sqrt{2 \cdot S(\omega_i) \cdot \delta\omega_i} \tag{4.26}$$

The wave elevation at an arbitrary location is the computed as:

$$\zeta_{Total} = \sum_{i=1}^{N} \left(\zeta_i(X,Y,t)\right) \tag{4.27}$$

The phase angles (ε_i) for each of the wave components are random numbers which are uniformly distributed between $(-\pi,\pi)$ and drawn independently for each frequency ω_i.

The hydrostatic pressure is computed as:

$$p_{hys} = -\rho \cdot g \cdot Z \tag{4.28}$$

The dynamic pressure is computed as:

$$p_D = \sum_{i=1}^{N} p_{D_i} \tag{4.29}$$

The total orbital velocity vector is given by:

$$\vec{V}_{Total} = \sum_{i=1}^{N} \vec{V}_i \tag{4.30}$$

The total orbital acceleration vector is given by

$$\dot{\vec{V}}_{Total} = \sum_{i=1}^{N} \dot{\vec{V}}_i \tag{4.31}$$

4.11 Short-Crested Irregular Seaway Kinematics

The kinematics for a short crested irregular seaway are computed in a similar manner to a long-crested seaway. A spreading function, G(β), is introduced in order to distribute the energy of each wave frequency over several directions. Two common spreading functions are the cosine squared and cosine to fourth functions.

Definition of parameters in spreading function

$$\begin{aligned} &\beta: && Wave\ \ component\ \ \ direction \\ &\beta_0: && Seaway\ \ \ direction \\ &\beta_A: && Spreading\ \ \ angle \\ &\beta_L: && Lower\ \ limit\ \ for\ beta = \beta_0 - \frac{\beta_A}{2} \\ &\beta_U: && Upper\ \ limit\ \ for\ beta = \beta_0 + \frac{\beta_A}{2} \end{aligned} \tag{4.32}$$

Cosine Squared Spreading Function:

$$\mathrm{G}(\beta) = \frac{2}{\beta_A} \cdot \cos^2\left((\beta - \beta_0) \cdot \frac{\pi}{\beta_A}\right) \qquad Where: \ \beta_L < \beta < \beta_U \tag{4.33}$$

Cosine to the Fourth Spreading Function:

$$\mathrm{G}(\beta) = \frac{8}{3} \cdot \frac{1}{\beta_A} \cdot \cos^4\left((\beta - \beta_0) \cdot \frac{\pi}{\beta_A}\right) \qquad Where: \ \beta_L < \beta < \beta_U \tag{4.34}$$

The spectral density then becomes:

$$\mathrm{S}(\omega,\beta) = \mathrm{S}(\omega) \cdot \mathrm{G}(\beta) \tag{4.35}$$

The amplitude for the wave components is given by:

$$a_{i,j} = \sqrt{2 \cdot \mathrm{S}(\omega_i, \beta_j) \cdot \delta\omega_i \cdot \mathrm{G}(\beta_j) \cdot \delta\beta_j} \tag{4.36}$$

The wave elevation at an arbitrary location is the computed as:

$$\zeta_{Total} = \sum_{i=1}^{N} \sum_{j=1}^{N_\beta} \left(\zeta_{i,j}(X,Y,t)\right) \tag{4.37}$$

The hydrostatic pressure is computed as:

$$p_{hys} = -\rho \cdot g \cdot Z \tag{4.38}$$

The dynamic pressure is computed as:

$$p_D = \sum_{i=1}^{N} \sum_{j=1}^{N_\beta} p_{D_{i,j}} \tag{4.39}$$

The total orbital velocity vector is given by:

$$\vec{V}_{Total} = \sum_{i=1}^{N} \sum_{j=1}^{N_\beta} \vec{V}_{i,j} \tag{4.40}$$

The total orbital acceleration vector is given by:

$$\dot{\vec{V}}_{Total} = \sum_{i=1}^{N} \sum_{j=1}^{N_\beta} \dot{\vec{V}}_{i,j} \tag{4.41}$$

4.12 Discretization of Spectra

The frequencies of the discretized spectrum may have equal or unequal spacing between them. In the case of unequal spacing, the frequencies are selected so that the area of each slice of the spectrum ($S(\omega_i)\cdot \delta\omega_i$) is constant. This yields wave component amplitudes that are constant across frequencies for a given wave direction β.

The spectra are truncated so that wave components with infinitesimal amplitudes or extremely high frequencies are excluded, as they do not contribute significantly to the overall result and add computational burden. The truncation limits are set so that significant wave height computed from the area under the discretized spectrum is less than 0.5%. A lower truncation limit, ω_{llim}, of $0.6\cdot\omega_m$ and an upper truncation limit, ω_{ulim}, of $4.0\cdot\omega_m$ are appropriate for all spectra defined in the Theoretical Spectra section.

4.13 Theoretical Spectra

There are several commonly used theoretical spectra that are useful in engineering analysis. The most widely used spectra for ship design evaluations are Bretschneider, JONSWAP, and Pierson-Moskowitz.

4.13.1 Pierson-Moskowitz – Input H_s only

Spectral definition:

$$\mathrm{S}(\omega) = \frac{0.0081\cdot g^2}{\omega^5}\cdot \exp\left(\frac{-0.032\cdot g^2}{H_s\cdot\omega^4}\right) \tag{4.42}$$

Modal period:

$$\omega_m = \sqrt{\frac{g}{H_s}} \tag{4.43}$$

4.13.2 Bretschneider – Input H_s and ω_m

Spectral definition:

$$S(\omega)=\frac{1.25}{4}\cdot\frac{\omega_m^4}{\omega^5}\cdot H_s^2\cdot\exp\left[-1.25\cdot\left(\frac{\omega_m}{\omega}\right)^4\right] \tag{4.44}$$

4.13.3 JONSWAP – Input H_s and ω_m

Spectral definition:

$$S(\omega)_{JONSWAP}=\frac{A}{\omega^5}\exp\left[-1.25\left(\frac{\omega_m}{\omega}\right)^4\right]\gamma^{\exp\left[-(\omega-\omega_m)^2/\left(2\sigma^2\omega_m^2\right)\right]} \tag{4.45}$$

where,

$$A=\frac{H_S^2}{16}\omega_m^4\left(0.06533\gamma^{0.8015}+0.13467\right)^{-1}$$
$$\sigma=\begin{cases}0.07 \ for\ \omega\le\omega_m\\ 0.09 \ for\ \omega>\omega_m\end{cases} \tag{4.46}$$
$$\gamma=3.3 \textit{(peaking factor)}$$

4.13.4 Ochi Hubble Six Parameter Spectrum and other Bimodal Spectra

Wave spectra obtained from measured data in the ocean often have double peaks (bimodal frequencies). The Ochi Hubble 6-Parameter spectral formulation as described in Ochi and Hubble (1976) is actually the sum of two three-parameter spectra, which in general can represent a spectrum with two peaks. The six input parameters are the modal frequency ω_m, significant wave height H_s, and peaking factor γ for each of the two spectra. Note that Ochi and Hubble (1976) use the symbol λ for the peaking factor instead of γ. Since λ is used for wavelength we have used γ, which is consistent with the symbol used for a peaking factor in the JONSWAP spectra. The spectral definition is:

$$S(\omega)=\frac{1}{4}\sum_{j=1,2}\frac{\left(\frac{4\gamma_j+1}{4}\omega_{mj}\right)^{\gamma_j}}{\Gamma(\gamma_j)}\frac{H_{sj}^2}{\omega^{4\gamma_j+1}}\exp\left[-\left(\frac{4\gamma_j+1}{4}\right)\left(\frac{\omega_{mj}}{\omega}\right)^4\right] \tag{4.47}$$

where Γ is the Gamma function. The total significant wave height for the Ochi Hubble Spectrum is:

$$H_s=\sqrt{H_{s1}^2+H_{s2}^2} \tag{4.48}$$

Each of the two spectra making up the total spectrum reduces to the Bretschneider spectrum when $\gamma_j = 1$, since $\Gamma(1) = 1$.

The Ochi Hubble six parameter spectrum is a common type of bimodal spectrum. Bimodal spectra can also be formed by summing any two single peak spectra; for example, a two peak JONSWAP spectrum could be formed as the sum of two JONSWAP spectra with different modal frequencies. Bimodal spectra can be used to represent a combination of a wind driven sea and a swell. Swell, which is composed of waves generated from an old storm traveling across the ocean, typically consists of long-crested waves with longer wave periods. The direction of the swell may be different than that of the wind driven sea. For the case of a bimodal spectrum composed of a wind driven sea and a swell, the two spectra forming the total spectrum may each have a separate primary wave direction, β_0, as well as a different spreading function G(β).

During the original storm large that creates swell conditions, wind driven waves are generated with the wave spectrum that probably resembles a fully developed Pierson-Moskowitz spectrum, consisting of many waves over a wide period range. As the waves travel away from the center of the storm, the longer waves will travel faster than the shorter waves, and several hundred miles from the location of the original storm the waves will have separated into groups with different wavelengths (note that wavelength is directly proportional to wave period). All of the waves will decrease in amplitude as they travel out of the storm area into areas with less wind, but the longer waves will retain more energy, and most of the shorter waves (periods < 12 seconds) will decay completely and die out within a few hundred miles of the original storm. The remaining longer waves will appear as a nearly long-crested, uniform wave train, which is what is referred to as swell. An observer a thousand miles or more from the original storm location will first see waves of smaller amplitude with longer periods of around 20 seconds. These are the waves from the low frequency end of the original storm wave spectrum that travel the fastest and reach the observer first. Over time the amplitude of the swell will increase and the period will decrease. For a typical swell the most wave energy is usually contained in waves with a 15-17 second period, corresponding roughly to the modal period of the original storm. As the period continues to decrease past 14 seconds the swell amplitude will likely decrease and the swell will eventually fade away.

4.14 Spectral Moments

The n^{th} moment of a seaway spectrum is calculated as follows:

$$m_n = \int_0^\infty \omega^n \cdot S(\omega) \cdot d\omega \tag{4.49}$$

The spectral moments are useful in the calculation of several parameters related to the seaway. If a narrow-banded, stationary, Gaussian process with zero mean is assumed, then the significant wave height is given by:

$$H_s = 4 \cdot \sqrt{m_0} \tag{4.50}$$

The average period of the seaway is given by:

$$T_{Avg} = 2 \cdot \pi \cdot \frac{m_0}{m_1} \tag{4.51}$$

The zero-crossing period of the seaway is given by:

$$T_0 = 2 \cdot \pi \cdot \sqrt{\frac{m_0}{m_2}} \tag{4.52}$$

4.15 3rd Order Stokes' Wave – Infinite-Depth

The equations listed in the previous sections apply to linear waves. In this section the equations for a monochromatic 2nd and 3rd order Stokes' wave in infinite water depth will be presented. Stokes' waves are based on nonlinear solutions for plane progressive waves derived from power series expansions in the wave amplitude. Stokes' waves of varying orders can be derived by retaining more terms in the expansion.

For a 2nd and 3rd order Stokes' wave in deep water, the dispersion relation becomes:

$$\omega^2 = gk\left(1 + k^2 a^2\right) \tag{4.53}$$

where wave number is defined as:

$$k = \frac{2 \cdot \pi}{\lambda} \tag{4.54}$$

Note that many of the simple relationships between the wavelength and the wave frequency that are valid for linear waves (Eqns. ((4.1)- (4.4)) were derived from the simpler linear dispersion relation, and cannot be used for higher order Stokes' waves. The relationships shown in Equations (4.54) through (4.56) are valid for a monochromatic Stokes' wave of any order.

Frequency:

$$\omega = 2 \cdot \pi \cdot f = \frac{2 \cdot \pi}{T} \tag{4.55}$$

Wave Celerity (Phase Velocity):

$$c = \frac{\omega}{k} \tag{4.56}$$

If the wavelength or wave number is specified, the wave frequency and period can be computed from Eqns. ((4.53)-(4.55)). If the frequency or period is specified, Eqn. (4.53) must be solved to obtain the wave number. This equation has two complex roots and only one purely real root. The solution for the wave number, *k*, for a specified frequency is:

$$k = \frac{\chi^{1/3}}{6ag} - \frac{2g}{a\chi^{1/3}}$$

where,

$$\chi = \left(108\omega^2 a + 12\sqrt{3}\sqrt{4g^2 + 27\omega^4 a^2}\right)g^2 \tag{4.57}$$

When the nonlinear dispersion relation, Eqn. (4.53), is used in place of the linear dispersion relation, the linear equations for the velocity potential, Eqns. (4.11) and (4.12), are accurate up to and including terms of order $(ka)^3$.

In order to simplify the equations, the following substitution will be made:

$$\Theta = \omega t - kX\cos(\beta) - kY\sin(\beta) + \varepsilon \tag{4.58}$$

The wave elevation for a 2nd order Stokes' wave is:

$$\zeta = a\cos(\Theta) + \frac{1}{2}ka^2\cos(2\Theta) \tag{4.59}$$

The wave elevation for a 3rd order Stokes' wave is:

$$\zeta = a\cos(\Theta) + \frac{1}{2}ka^2\cos(2\Theta) + \frac{3}{8}k^2a^3\cos(3\Theta) \tag{4.60}$$

For linear and 2nd order Stokes' waves, the peak to trough wave height, *H*, is equal to 2*A*. For a 3rd order Stokes' wave, the wave height is expressed as:

$$H = 2a + \frac{3}{4}k^2a^3 \tag{4.61}$$

Note that for anything other than linear waves, the amplitude, *A*, no longer relates to a physical dimension, such as the distance from the mean water level to the peak or trough.

Since the velocity potential used for the linear wave is valid through third order when the wave number and frequency satisfy the nonlinear dispersion relation in Eqn. (4.53), the wave orbital velocity can be obtained by differentiating the formula for the velocity potential. The equations in (4.16) use the linear dispersion relation and are valid only for linear calculations. The correct equations for the orbital velocity accurate up to and including terms of order $(ka)^3$ are given in Equations (4.62) below:

$$\begin{aligned}
&\vec{V} = \{u_w, v_w, w_w\} = \nabla\Re(\varphi) = \left(\frac{\partial}{\partial X}\Re(\varphi), \frac{\partial}{\partial Y}\Re(\varphi), \frac{\partial}{\partial Z}\Re(\varphi)\right) \\
&u_w = \frac{a\cdot g\cdot k}{\omega}\cdot e^{k\cdot Z}\cdot\cos(\omega\cdot t - k\cdot X\cdot\cos(\beta) - k\cdot Y\cdot\sin(\beta) + \varepsilon)\cdot\cos(\beta) \\
&v_w = \frac{a\cdot g\cdot k}{\omega}\cdot e^{k\cdot Z}\cdot\cos(\omega\cdot t - k\cdot X\cdot\cos(\beta) - k\cdot Y\cdot\sin(\beta) + \varepsilon)\cdot\sin(\beta) \\
&w_w = \frac{-a\cdot g\cdot k}{\omega}\cdot e^{k\cdot Z}\cdot\sin(\omega\cdot t - k\cdot X\cdot\cos(\beta) - k\cdot Y\cdot\sin(\beta) + \varepsilon)
\end{aligned} \tag{4.62}$$

The pressure at any point under the wave can be computed from the Bernoulli Equation. In deriving the first order equation for dynamic pressure, Eqn. (4.15), the $|\vec{V}|^2$ term in the Bernoulli Equation was not included as this was of the order $(ka)^2$. For 2nd and 3rd order Stokes' waves this term must be retained. The total incident wave pressure for a 2nd and 3rd order Stokes' wave, including both the hydrostatic and dynamic components is:

$$p = -\rho gZ + \rho gae^{kZ}\cos(\Theta) - \frac{1}{2}\rho\left(\frac{gak}{\omega}\right)^2 e^{2kZ} \tag{4.63}$$

The total pressure specified by the above equation approaches zero at the water surface, so it is not necessary to apply the pressure stretching used in the linear wave theory. Since the formulae for the velocity potential of a monochromatic 2nd and 3rd order Stokes' wave are identical, and do not include terms of $O(ka)^2$ or higher, the expressions for the wave orbital velocity and pressure given in Equations (4.62) and (4.63) are the same for both 2nd and 3rd order Stokes' waves. Only the expressions for wave elevation are different between the 2nd and 3rd order Stokes' waves.

4.16 Inputs and Defaults

Tempest should be able to use an arbitrary number of linear seaways. There are three ways to specify the input parameters for a seaway and each seaway should be able to use a different method. The input methods and their associated options are:

- Regular wave inputs
  - Required:
    - Wave amplitude: A
    - Either period, length or frequency: T, λ, or ω
  - Optional:
    - ε – default: 0 deg
    - Order of Stokes' wave – default: linear
- Selection of a theoretical spectrum
  - Required: Spectrum type, H_s, T_m, N
  - Optional:
    - β_0 – default: 0 deg
    - ω_{llim} – default: $0.6 \cdot \omega_m$
    - ω_{ulim} – default: $4.0 \cdot \omega_m$
    - r_ε (random seed for automatically selected phase angles) – default: random
    - Pressure stretching flag – default: use pressure stretching
    - Frequency spacing method – default: constant area
      - Options are constant $\delta\omega$ or constant area
    - Spreading function – default: no spreading
      - Spreading function type – default: cosine squared
      - β_A – default: 90 degrees
      - N_β – default: 5 (not including end points of spreading function – they are 0)
- Input of spectral ordinates
  - Required: ω_i, $S(\omega_i)$
  - Optional:
    - β_i – default: 0 deg
    - ε_i – default: random
      - mutually exclusive with r_ε
    - r_ε – default: random
      - mutually exclusive with ε_i
    - Pressure stretching flag – default: use pressure stretching
- Input of wave components
  - Required: ω_i, a_i
  - Optional:

    - β_i – default: 0 deg
    - ε_i – default: random
        - mutually exclusive with r_ε
    - r_ε – default: random
        - mutually exclusive with ε_i
    - Pressure stretching flag – default: use pressure stretching

4.17 Intermediate Output

Tempest shall report, in some manner, H_s computed from the discretized spectrum. The graphical Tempest client shall have the ability to display an autocorrelation function for the discretized spectrum computed by a discrete cosine transform of the spectrum.

5 Autopilot Control

Tempest Level-0 will include a simple PID control algorithm to control the rudder deflection and propeller RPM. The rudder control algorithm will be set up to deflect the rudder in an unsteady manner to maintain a constant heading angle specified by the user. In the Level-0 theory, the propeller RPM is assumed to be constant, but a PID control algorithm is included for the propeller to help compute the propeller RPM required to achieve a desired speed in calm water.

The PID controller for the rudder requires the following user-supplied inputs:

- ψ_D – desired heading angle
- G_p – gain coefficient for proportional term
- G_i – gain coefficient for integral term
- G_d – gain coefficient for derivative term
- δ_{max} – maximum rudder deflection angle
- $\dot{\delta}$ – rudder slew rate
- P_B – Proportional band of controller
- I_{max} – upper limit on the amplitude of the integral error

The commanded rudder deflection angle at time t is computed as:

$$\delta_c = \delta_0 + G_p\left(\psi_D - \psi\right) + G_i \int_0^t \left(\psi_D - \psi(\tau)\right) d\tau + G_d \dot{\psi} \quad , \tag{5.1}$$

where $\dot{\psi}$ is the time derivative of the ship heading, which is computed using finite differencing of the ship heading angle at the current and previous time step. The integral gain term above is limited by the user-supplied value, I_{max}:

$$\left| \int_0^t \left(\psi_D - \psi(\tau)\right) d\tau \right| \le I_{\max} \quad . \tag{5.2}$$

To compute the actual rudder deflection angle at each time step, it is assumed that the rudder servo is a linear first-order lag with slew rate, $\dot{\delta}$, and proportional band, P_B. The actual rudder deflection is then computed as:

$$\delta = \delta_0 + \left(1 - \exp\left\{\frac{\Delta t\ \dot{\delta}}{P_B}\right\}\right)\left(\delta_c - \delta_0\right) \quad , \tag{5.3}$$

where δ_0 is the rudder angle at the beginning of the time step and the second term is the increment of the rudder angle. The increment is limited by the rudder slew rate:

$$\left|\frac{\left(\delta_c - \delta_0\right)}{\Delta t}\right| \le \dot{\delta} \quad , \tag{5.4}$$

and the total rudder deflection is limited by the maximum rudder angle, δ_{max}:

$$\left|\delta\right| \le \delta_{\max} \quad . \tag{5.5}$$

6 Forces

Tempest requires the total force and moment vector to be computed at each time step in order to solve the equations of motion for the ship. In the Level-0 theory the forces and moments are referenced to the center of gravity of the ship. In this document, the term “forces” will hereafter be assumed to include both forces and moments. In Tempest it will be assumed that the total force can be decomposed into various force contributions, and the total forces on the hull can therefore be obtained by summing the various force contributions. In the Level-0 theory, the following force contributions will be considered:

- Hydrostatic forces – F_{hys}
- Froude-Krylov forces – F_{FK}
- Radiation forces – F_{rad}
- Diffraction forces – F_{dif}
- Hull resistance – F_{resist}
- Bilge keel forces – F_{BK}
- Bare hull maneuvering forces (includes skeg force) – F_{man}
- Rudder and fin forces – F_R
- Propulsion forces – F_P
- Weight of ship – W

The force from the weight of the ship is described along with the equations of motion in Section 3.3. The remaining force components are discussed in Sections 6.1 through 6.7. As the total force is computed as the linear superposition of component forces, special care is taken to avoid either missing important force components or the double counting forces. An example of double counting forces would be low frequency radiation forces and maneuvering forces.

Another example would be computing and including the rudder force when the hull maneuvering force is based on empirical data that includes the rudder force already. The areas where double counting of forces is a concern and how double counting is avoided is discussed in the sections describing each force component.

The total force in the ship-fixed reference frame acting on the ship at the center of gravity of the ship is the sum of the component forces.

$$\vec{F}_{total} = \vec{F}_{hys} + \vec{F}_{FK} + \vec{F}_{rad} + \vec{F}_{dif} + \vec{F}_{resist} + \vec{F}_{BK} + \vec{F}_{man} + \vec{F}_{R} + \vec{F}_{P} + \vec{W} \quad (6.1)$$

Note that in the Level-0 equations of motion the weight of the ship is already accounted for and the added mass terms in the radiation force are moved to the left hand side. Therefore the force vector used to form the right hand side of the equations of motion described by Equation (3.27) is computed as:

$$\vec{F}_{RHS} = \vec{F}_{hys} + \vec{F}_{FK} + \vec{F}_{rad\_damping} + \vec{F}_{dif} + \vec{F}_{resist} + \vec{F}_{BK} + \vec{F}_{man} + \vec{F}_{R} + \vec{F}_{P} + \vec{W} \quad (6.2)$$

The terms forming the right hand side of Equation (6.2) are defined by Equations (6.3), (6.16), (6.23), (6.28), (14b), (6.72), (6.103) and (6.123).

6.1 Hydrostatic and Froude-Krylov Force

The hydrostatic pressure force on the hull is computed by integrating the hydrostatic pressure over the wetted hull surface. The Froude-Krylov force is computed by integrating the dynamic wave pressure over the wetted hull surface. In both cases the wetted hull surface is defined as the hull surface below the incident wave surface. In other words, the diffracted wave is not considered in computing the hydrostatic or Froude-Krylov force on the hull in the Level-0 theory. The combined hydrostatic Froude-Krylov force is the integration over the ship's surface of the total pressure that would exist in the incident wave field in the absence of the ship.

In Tempest, the ship hull geometry will be described as a meshed surface defined by the vertices that define the mesh. The first step is to compute the total wave height near the ship in order to determine which vertices are wet and which are dry. The total wave height at any location at an instant in time can be computed from Equation (4.27). By computing the total wave height at each vertex in the mesh, the intersection of the incident wave surface with the ship hull can be determined. The next step is to compute the hydrostatic pressure, p_{hys}, and the dynamic wave pressure, p_D, at each vertex at the current time step. The hydrostatic pressure at any point is computed using Eqn. (4.28). The dynamic pressure at any point at a given instance in time is computed using Eqn. (4.29). When computing the dynamic pressure, Wheeler's method of pressure stretching is applied as described in Section 4.8 to ensure that the total pressure on the water surface goes to zero. The combined hydrostatic and Froude-Krylov force is then computed as the integral of the hydrostatic and Froude-Krylov pressures over the wetted hull surface.

$$F_{hys+FK} = \int_{S_{wet}} \left(p_{hys} + p_D \right) dS \quad (6.3)$$

The integral shown in Equation (6.3) is evaluated numerically using the meshed hull surface and the values of the hydrostatic and dynamic pressure computed at each vertex.

6.2 Hydrodynamic Perturbation Force

The total force acting on the ship will be computed by linear superposition of force contributions from various sources. The hydrodynamic perturbation forces are decomposed into radiation forces and wave exciting forces. The radiation forces represent the forces due to the oscillating motion of the ship in calm water. The wave exciting forces are further broken down into the Froude-Krylov forces and the diffraction forces. The Froude-Krylov forces, which are described in Section 6.1, are the component of the wave exciting force resulting from the integration over the ship's surface of the incident wave pressure field that would exist in the absence of the ship. The diffraction forces represent the force due to the diffraction of the incident wave due to the presence of the ship.

6.2.1 Radiation Force

6.2.1.1 Summary of Theory

This section describes the theory for computing the radiation forces in the Level-0 implementation of the time domain dynamic stability code Tempest. The Level-0 Tempest is intended to be a code based on simple theories that will provide an initial framework into which more accurate theories can be implemented. In the Level-0 theory, the radiation force at each time step will be computed from the added mass and damping coefficients interpolated at the instantaneous encounter frequency, and the "memory" force relating to the radiation force will be ignored.

The total force acting on the ship will be computed by linear superposition of the forces contributed from various sources. The seakeeping forces are decomposed into radiation forces and wave exciting forces. The radiation forces represent the forces acting on the ship as it undergoes prescribed oscillatory motions in calm water. The radiation forces are further decomposed by examining the oscillatory motion in each of the six degrees of freedom separately. For steady-state oscillatory motion at a single frequency, the hydrodynamic radiation force acting on the ship can be expressed as:

$$Frad_i(t) = \mathrm{Re}\left\{ e^{i\omega t} \sum_{j=1}^{6} \xi_j T_{ij} \right\} \tag{6.4}$$

where, ξ_j is the complex amplitude of the oscillation in the j^{th} mode, and T_{ij} is the complex transfer function which represents the force in the i^{th} direction due to a unit amplitude oscillation in the j^{th} mode, which can be decomposed into its real and imaginary parts:

$$T_{ij} = \omega^2 A_{ij} - i\omega B_{ij} \tag{6.5}$$

where A_{ij} and B_{ij} are real coefficients with A_{ij} being the added mass in the i^{th} mode due to unit acceleration in the j^{th} direction and B_{ij} being the damping coefficient in the i^{th} mode due to unit velocity in the j^{th} direction. The coefficients A_{ij} and B_{ij} are in general dependent upon the frequency of oscillation and the forward speed of the ship.

As the ship undergoes forced oscillation it will generate waves that will radiate outwards on the free surface. When the oscillatory motion is unsteady or has not reached steady state, the presence of the non-uniform radiated waves on the free surface results in a time dependent radiation force. This radiation force depends on the previous motion history of the ship, as the waves produced by the ship will continue to influence the pressure in the fluid and consequently the force on the ship, as they radiate away. When the solution is computed in the time domain, this "memory" force is represented by the convolution integral in the formula below.

$$Frad_i(t) = \sum_{j=1}^{6} \left[-A_{ij\infty}\ddot{x}_j - B_{ij\infty}\dot{x}_j - \int_{-\infty}^{t} K_{ij}(t-\tau)\dot{x}_j(\tau)d\tau \right] \tag{6.6}$$

The convolution in the above equation can be expressed in terms of either the velocities $\dot{x}_j$ or the accelerations $\ddot{x}_j$:

$$\int_{-\infty}^{t} K_{ij}(t-\tau)\dot{x}_j(\tau)d\tau = \int_{-\infty}^{t} L_{ij}(t-\tau)\ddot{x}_j(\tau)d\tau \tag{6.7}$$

where the impulse response functions used for the kernels are related to each other by differentiation:

$$K_{ij}(t) = \frac{\partial}{\partial t} L_{ij}(t) \tag{6.8}$$

The impulse response functions can be computed from the frequency domain added mass and damping coefficients by Fourier sine or cosine transforms as:

$$\begin{aligned} L_{ij}(t) &= \frac{2}{\pi}\int_0^\infty \left[A_{ij}(\omega) - A_{ij}(\infty)\right]\cos(\omega t)d\omega &&= \frac{2}{\pi}\int_0^\infty \frac{B_{ij}(\omega)}{\omega}\sin(\omega t)d\omega \\ K_{ij}(t) &= \frac{2}{\pi}\int_0^\infty \left[B_{ij}(\omega) - B_{ij}(\infty)\right]\cos(\omega t)d\omega &&= \frac{2}{\pi}\int_0^\infty \omega\left[A_{ij}(\omega) - A(\infty)\right]\sin(\omega t)d\omega \end{aligned} \tag{6.9}$$

In the Level-0 implementation of Tempest the "memory" force term in Equation (6.6) will be ignored. The user will input values for A_{ij} and B_{ij} over a range of frequencies at zero speed. The code will then interpolate to determine the values of A_{ij} and B_{ij} corresponding to the encounter frequency based on a time averaged value of the ship speed, the instantaneous heading, and the modal frequency of the incident wave spectrum. Setting the "memory" force to zero is equivalent to assuming that the added mass and damping coefficients do not vary with frequency. Based on this assumption, $A_{ij}(\omega) - A(\infty) = 0$ and $B_{ij}(\omega) - B(\infty) = 0$, the memory force term in the expression for the radiation force will be zero. In the simplified Level-0 theory, the expression for the radiation force becomes:

$$Frad_i(t) = \sum_{j=1}^{6} \left[-A_{ij}^{*}\ddot{x}_j - B_{ij}^{*}\dot{x}_j \right] \tag{6.10}$$

where A_{ij}^{*} and B_{ij}^{*} are the values of $A_{ij}(\omega)$ and $B_{ij}(\omega)$ interpolated at the encounter frequency.

In the linear hydrodynamic theory used for Level-0, the added mass and damping coefficients, A_{ij} and B_{ij}, are computed with the ship in an upright position on the calm free surface, with the assumption that the motions are small. This results in a paradox when applied to cases where the ship motions result in large pitch and/or roll angles. Consider a case where the ship roll angle is close to 90° and the ship is forced to oscillate in the ship-fixed z-direction. In this case the ship is "heaving" with respect to the ship-fixed axis system but "swaying" with respect to the upright axis system. If A_{33} and B_{33} are used to compute the force in ship-fixed z-direction, the force will be based on calculations made with an upright ship oscillating vertically on the free surface. If A_{22} and B_{22} are used, the force will be based on calculations made with the upright ship oscillating horizontally on the free surface. When applying linear theory to a case with a large pitch and/or roll angle, a choice must be made whether to use coefficients computed with approximately the correct geometry, but with the incorrect motion relative to the free surface, or to use coefficients computed with an incorrect geometry, but with the correct motion relative to the free surface. For consistency with the ship-fixed mass matrix, the A_{ij} and B_{ij} tensors are transformed from the upright coordinate system (yawed earth-fixed) to a ship-fixed coordinate system, which is applied in the Level-0 implementation of Tempest.

6.2.1.2 Input

The user will provide a set of added mass and damping coefficients for the ship at zero speed. Only the non-zero coefficients listed at the end of this section will be provided, and these will be provided over a sufficient range of frequencies to allow the program to interpolate values at the instantaneous encounter frequency. The transfer functions should be defined in the coordinate system with x forwards, y to port and z upwards with the origin at the ship center of gravity. If a different origin is used to define the transfer functions, the origin location must also be specified as input and Tempest will perform a transformation so the moments are referenced to the center of gravity. A wave frequency corresponding to the modal period will also be specified for computing the encounter frequency. The Level-0 theory will assume that the ship is symmetric port and starboard. With this assumption the motions and forces in the lateral plane and vertical plane can be considered separately, and any term with one odd index and one even index will be zero.

$$
\begin{aligned}
&A_{12}^0 = A_{14}^0 = A_{16}^0 = A_{21}^0 = A_{23}^0 = A_{25}^0 = A_{32}^0 = A_{34}^0 = A_{36}^0 = 0 \\
&A_{41}^0 = A_{43}^0 = A_{45}^0 = A_{52}^0 = A_{54}^0 = A_{56}^0 = A_{61}^0 = A_{63}^0 = A_{65}^0 = 0 \\
&B_{12}^0 = B_{14}^0 = B_{16}^0 = B_{21}^0 = B_{23}^0 = B_{25}^0 = B_{32}^0 = B_{34}^0 = B_{36}^0 = 0 \\
&B_{41}^0 = B_{43}^0 = B_{45}^0 = B_{52}^0 = B_{54}^0 = B_{56}^0 = B_{61}^0 = B_{63}^0 = B_{65}^0 = 0
\end{aligned}
\tag{6.11}
$$

The “0” superscript is used to indicate zero speed. At zero speed, the added mass and damping tensors are symmetric:

$$
A_{ij}^{\,0} = A_{ji}^{\,0},\; B_{ij}^{\,0} = B_{ji}^{\,0}. \tag{6.12}
$$

This results in only 12 unique non-zero added mass and damping coefficients that the user must provide as input. These coefficients can be obtained from either a strip theory code or from a panel method such as WAMIT. In order to interpolate values of each coefficient at the encounter frequency, the user will need to compute the values over a range of frequencies covering the range of expected encounter frequencies. The coefficients should be provided as dimensional values with the units as specified in Table 6-1:

Table 6-1 Units for Added Mass and Damping Coefficients

i	j	Units for A_{ij}	Units for B_{ij}
1,2,3	1,2,3	kg	kg/sec
1,2,3	4,5,6	kg m	kg m/sec
4,5,6	1,2,3	kg m	kg m/sec
4,5,6	4,5,6	kg m^2	kg m^2/sec

The following is a list of the 3-D, zero-speed added mass and damping coefficients that the user will provide as input:

$$A_{11}^0, A_{13}^0, A_{15}^0, A_{22}^0, A_{24}^0, A_{26}^0, A_{33}^0, A_{35}^0, A_{44}^0, A_{46}^0, A_{55}^0, A_{66}^0$$

$$B_{11}^0, B_{13}^0, B_{15}^0, B_{22}^0, B_{24}^0, B_{26}^0, B_{33}^0, B_{35}^0, B_{44}^0, B_{46}^0, B_{55}^0, B_{66}^0$$

6.2.1.3 Implementation

The zero speed added mass and damping coefficients will be provided over a range of frequencies as input to the program. At each time step the program will interpolate each A_{ij} and B_{ij} coefficient to obtain a value corresponding to the encounter frequency based on the modal frequency of the incident wave. The interpolated values will then be corrected to account for the forward speed of the ship. In calm water, the zero frequency value of A_{ij} should be used and B_{ij} should be set to zero. No forward speed correction is applied to A_{ij} in calm water.

The A_{ij} and B_{ij} coefficient matrices are transformed from an upright coordinate system to the ship-fixed coordinate system. The added mass terms will be applied to the left hand side of the equations of motion as described in Section 3.5. The force from the damping terms will be computed and added as a component of the forces included on the right hand side of the equations of motion.

The formulae used to correct the added mass and damping coefficients for forward speed were taken from Principles of Naval Architecture (PNA 1989). These formulae are derived from the strip theory outlined in Salvensen, Tuck, and Faltinsen (1970). The formulae in PNA are modified from those in the original Salvensen et. al. (1970) work, in that the original theory included transom stern corrections that have been omitted and the original work did not include the surge degree of freedom, which has been added. The following terms do not have forward speed corrections and will be set to their zero speed values:

$$A_{13} = A_{31} = A_{13}^0$$

$$B_{13} = B_{31} = B_{13}^0$$

$$A_{24} = A_{42} = A_{24}^0$$

$$B_{24} = B_{42} = B_{24}^0$$

$$A_{11} = A_{11}^0$$

$$B_{11} = B_{11}^0$$

$$A_{22} = A_{22}^0$$

$$B_{22} = B_{22}^0$$

$$A_{33} = A_{33}^0$$

$$B_{33} = B_{33}^0$$
$$A_{44} = A_{44}^0$$
$$B_{44} = B_{44}^0$$

The remaining terms will be corrected to account for forward speed using the formulae listed in Table 6-2 below:

Table 6-2 Forward speed corrections for added mass and damping coefficients.

Vertical Plane	Lateral Plane
$A_{15} = A_{15}^0 - \frac{U_0}{\omega_e^2} B_{13}$	$A_{26} = A_{26}^0 + \frac{U_0}{\omega_e^2} B_{22}$
$A_{51} = A_{15}^0 + \frac{U_0}{\omega_e^2} B_{13}$	$A_{62} = A_{26}^0 - \frac{U_0}{\omega_e^2} B_{22}$
$B_{15} = B_{15}^0 + U_0 A_{13}$	$B_{26} = B_{26}^0 - U_0 A_{22}$
$B_{51} = B_{15}^0 - U_0 A_{13}$	$B_{62} = B_{26}^0 + U_0 A_{22}$
$A_{35} = A_{35}^0 - \frac{U_0}{\omega_e^2} B_{33}$	$A_{46} = A_{46}^0 + \frac{U_0}{\omega_e^2} B_{24}$
$A_{53} = A_{35}^0 + \frac{U_0}{\omega_e^2} B_{33}$	$A_{64} = A_{46}^0 - \frac{U_0}{\omega_e^2} B_{24}$
$B_{35} = B_{35}^0 + U_0 A_{33}$	$B_{46} = B_{46}^0 - U_0 A_{24}$
$B_{53} = B_{35}^0 - U_0 A_{33}$	$B_{64} = B_{46}^0 + U_0 A_{24}$
$A_{55} = A_{55}^0 + \frac{U_0^2}{\omega_e^2} A_{33}$	$A_{66} = A_{66}^0 + \frac{U_0^2}{\omega_e^2} A_{22}$
$B_{55} = B_{55}^0 + \frac{U_0^2}{\omega_e^2} B_{33}$	$B_{66} = B_{66}^0 + \frac{U_0^2}{\omega_e^2} B_{22}$

U_0 is the mean forward speed of the ship. In Tempest (Level-0) U_0 will be computed as the running average of the instantaneous velocity of the ship along the ship-fixed x-axis over an interval of time before the current time step. The size of the time interval used for the averaging will be an optional user input, and the default size of the time interval will be 15 seconds. ω_e is the encounter frequency, which is computed using the formula below:

$$\omega_e = \omega - \frac{\omega^2}{g} U_0 \cos\mu \tag{6.13}$$

The value of ω used to compute the encounter frequency will be the frequency of the incident wave if a regular wave is specified and the modal frequency if an irregular wave spectrum is specified. The angle μ is the heading angle. It will be computed in Tempest at each time step as the angle between the ship-fixed x-axis and the primary direction of the waves, with 180° corresponding to head seas and 0° to following seas. In calm water cases, the encounter frequency should be set to zero and the forward speed corrections should not be included. Also in calm water the damping terms, B_{ij}, should all be set to zero.

The code will then transform the added mass and damping terms from an upright system to a ship-fixed system. The transformation matrix for converting from the yawed upright system to the ship-fixed system is:

$$[T_{U/S}] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{6.14}$$

where ϕ is the roll angle and θ is the pitch angle, as defined in Section 3.2. In order to rotate the 6 x 6 added mass and damping matrices, each 3 x 3 quadrant of the matrix is transformed separately as follows:

$$\begin{bmatrix} A_{11}' & A_{12}' & A_{13}' \\ A_{21}' & A_{22}' & A_{23}' \\ A_{31}' & A_{32}' & A_{33}' \end{bmatrix} = [T_{U/S}] \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} [T_{U/S}]^T \tag{6.15}$$

Note that prime notation is used in the case of the added mass and damping coefficients to denote values in the ship-fixed coordinate system. Throughout this document, prime notation generally refers to the yawed upright frame of reference. However, the user supplied-coefficients, A_{ij} and B_{ij}, calculated in the yawed upright frame are conventionally referenced without prime notation. This inconsistency in notation in this document is a result of maintaining conventional notation for the initially-calculated added mass and damping coefficients.

After transforming the matrices for both the added mass and damping coefficients, the transformed 6x6 added mass matrix will be added to the left hand side of the equations of motion in place of the infinite frequency added mass terms discussed in Section 3.5. The radiation forces due to the damping will be computed using the equation listed below and added to forces composing the right hand side of the equations of motion.

$$Frad\_damping_i(t) = \sum_{j=1}^{6} - B_{ij}'\left(\dot{x}_j - \bar{\dot{x}}_j\right) \tag{6.16}$$

where, $\bar{\dot{x}}_j$ is the mean velocity in the j^{th} direction. The values for $\bar{\dot{x}}_j$ will be computed as the running average of the velocity vector over the time interval before the current time step. The length of the time interval will be the same used to compute U_0, and $\bar{\dot{x}}_1$ will be equal to U_0. At the start of the simulation the averaging will take place from $t = 0$ seconds to the current time, until the total simulation time exceeds the length of the specified time integral.

If it is desired to list the total ship-fixed radiation force as an output at each time step, this can be computed with the following equation:

$$Frad_i(t) = \sum_{j=1}^{6}\left[-A_{ij}'\ddot{x}_j - B_{ij}'\left(\dot{x}_j - \dot{\bar{x}}_j\right)\right] \tag{6.17}$$

6.2.2 Diffraction Force

6.2.2.1 Summary of Theory

This section describes the theory for computing the diffraction forces in the Level-0 implementation of the time domain dynamic stability code Tempest. The Level-0 Tempest is intended to be a code-based on simple theories that will provide an initial framework into which more accurate theories can be implemented.

The diffraction force in the Level-0 Tempest code will be calculated from a set of pre-computed complex transfer functions. The transfer functions will be provided for a range of wave frequencies and heading angles. The transfer functions are also dependent on the speed of the ship. The influence of forward speed will be included either through the user supplying a set of transfer functions for several speeds and interpolating to determine coefficients at the instantaneous speed at each time step, or by applying forward speed corrections to coefficients supplied by the user only at zero speed. The real part of the complex transfer function represents the component of the force that is in phase with the wave, while the imaginary part of the complex transfer function represents out of phase component. It is assumed that the waves are defined as the linear superposition of long-crested waves traveling in the direction of the earth-fixed X-axis. The incident wave height at the center of gravity of the ship is then defined by the expression:

$$\varsigma(X,t) = \sum_{n=1}^{N}\varsigma_n \sin\left(\omega_n t - k_n X + \varepsilon_n\right) \tag{6.18}$$

where there are N wave components. ω_n, k_n, ε_n, ζ_n are, respectively, the frequency, wave number, phase and amplitude of wave component n. Details of computing the incident wave height for cases where the wave components do not all travel along the earth-fixed X-axis are given in Section 4.11. If a wave component is traveling at an angle β to the earth-fixed X-axis, $(-k_nX)$ should be replaced with $(-k_nX\cos\beta - k_nY\sin\beta)$ in all of the equations found in this section, where X and Y are the positions of the center of gravity of the ship with respect to the earth-fixed origin in the earth-fixed X-direction.

The total diffraction force in the i^{th} direction will be the sum of the diffraction force contributions from each wave component defining the wave system.

$$Fdif_i(t) = \sum_{n=1}^{N}\left\{\mathrm{Re}\left[F_i^D(\omega_n,\mu)\right]\varsigma_n \sin\left(\omega_n t - k_n X + \varepsilon_n\right) + \mathrm{Im}\left[F_i^D(\omega_n,\mu)\right]\varsigma_n \cos\left(\omega_n t - k_n X + \varepsilon_n\right)\right\} \tag{6.19}$$

The complex diffraction transfer function for the force in the i^{th} direction is expressed as a function of frequency and heading angle:

$$F_i^D(\omega_n,\mu) = \mathrm{Re}\left[F_i^D(\omega_n,\mu)\right] + i\,\mathrm{Im}\left[F_i^D(\omega_n,\mu)\right] \tag{6.20}$$

In the linear seakeeping theory used for Level-0, the diffraction force transfer function coefficients are computed with the ship in an upright position on the mean free surface, with the assumption that the wave amplitudes are small. The resulting forces will be computed in a yawed upright coordinate system with origin at the center of gravity. These forces must be transformed to the ship-fixed coordinate system with origin at the center of gravity before they are added to the other component forces.

6.2.2.2 Input

The user will provide a set of diffraction force complex transfer function coefficients computed for the ship either at zero speed or for a range of speeds. The set of transfer functions must cover a full range of headings and wave frequencies. The transfer functions should provide forces and moments about an origin at the center of gravity of the ship. The transfer function coefficients should be provided as dimensional values with the units specified in Table 6-3:

Table 6-3 Units for Diffraction Transfer Functions

i	Units for F^{D}_{i}
1,2,3	N/m
4,5,6	N m/ m

The transfer functions should be defined in the coordinate system with x forwards, y to port, and z upwards with the origin at the ship center of gravity. If a different origin is used to define the transfer functions, the origin location will be specified as input and Tempest will perform a transformation so the moments are referenced to the center of gravity.

6.2.2.3 Implementation

The diffraction force coefficients will be interpolated twice to obtain transfer functions at the current wave heading for each wave frequency used to make up the incident wave spectrum. The first interpolation will be to obtain a set of transfer functions at each of the wave frequencies used by the incident wave spectrum. The set of interpolated transfer functions will be a function of the input wave heading angles. Since the wave frequencies of the component waves forming the wave spectrum are constant, this first interpolation can be performed prior to the start of the time stepping procedure, with the interpolated values being stored and used at each time step. The second interpolation will be performed at each time step. The second interpolation will derive the transfer function at the current wave heading angle at every wave frequency used in the wave spectrum.

After the interpolation, the diffraction transfer functions will be corrected to account for the forward speed of the ship. The formulae used to correct the complex diffraction transfer functions for forward speed follow the formulae used in SMP and as described in Appendix B of Meyers, et. al. (1981). Forward speed corrections are only applied to the pitch and yaw moment coefficients:

$$\begin{aligned} F_1^D &= F_1^{D0} \\ F_2^D &= F_2^{D0} \\ F_3^D &= F_3^{D0} \\ F_4^D &= F_4^{D0} \end{aligned} \tag{6.21}$$

$$F_5^D = F_5^{D0} - \left(\frac{U_0}{i\omega_e}\right) F_3^{D0}$$

$$F_6^D = F_6^{D0} + \left(\frac{U_0}{i\omega_e}\right) F_2^{D0}$$

U_0 is the mean forward speed of the ship. In Tempest Level-0 it will be set to the the running average of the instantaneous velocity of the ship along the ship-fixed x-axis. ω_e is the encounter frequency, which is computed using the formula below:

$$\omega_e = \omega - \frac{\omega^2}{g} U_0 \cos\mu \tag{6.22}$$

The value of ω_e will be computed separately for each component wave frequency in the wave spectrum. The angle μ is the heading angle. It will be computed in Tempest at each time step as the angle between the ship-fixed x-axis and the primary direction of the waves, with 180° corresponding to head seas and 0° to following seas.

Alternatively, the user may input coefficients for both the radiation and diffraction forces at several speeds. Coefficients including forward speed can be computed by programs such as AQWA, FD-Waveload, and Precal. With this option the forward speed corrections will not be applied, but an additional interpolation will be required to determine the coefficients applicable to the instantaneous speed of the ship at each time step.

After the coefficients are interpolated and corrected for forward speed, the diffraction force can be computed.

$$Fdif_i(t) = \sum_{n=1}^{N} \left\{ \mathrm{Re}\left[F_i^D(\omega_n,\mu)\right] \varsigma_n \sin(\omega_n t - k_n X + \varepsilon_n) + \mathrm{Im}\left[F_i^D(\omega_n,\mu)\right] \varsigma_n \cos(\omega_n t - k_n X + \varepsilon_n) \right\} \tag{6.23}$$

The diffraction forces and moments will then be transformed from the yawed upright coordinate system to the ship-fixed coordinate system. The diffraction forces are then added to forces composing the right hand side of the equations of motion.

6.3 Resistance (Axial)

6.3.1 Summary

This section presents a method for computing the drag force on a ship in waves (or calm water) to be implemented in the time domain dynamic stability code Tempest. This method is appropriate for Theory Level-0 through Level-II. The Level-III theory will capture certain elements (wave making drag in particular) of the method presented herein. The last sub-section of will cover necessary changes when the Level-III Theory is implemented

6.3.2 Input

The user shall supply a curve of total resistance for the ship defined by several points. Let R_T be the total resistance of the ship and V be ship speed. The total resistance coefficient, C_T, is computed according to the following equation:

$$C_T = \frac{R_T}{0.5\rho S_{CW} V^2} \tag{6.24}$$

where:

C_T = total resistance coefficient
R_T = total ship resistance
V = ship speed in m/s
ρ = water density
S_{CW} = wetted surface of the ship in calm water

6.3.3 Scaling

If the simulation is to be performed at a scale other than that for which the resistance curve is given, then the frictional resistance coefficient of the ship is computed at each input speed according the ITTC 1957 guideline as:

$$C_\text{f} = \frac{0.075}{\left(\text{Log}_{10}(\text{Rn}) - 2\right)^2} \tag{6.25}$$

Where:
C_f = the frictional resistance coefficient
Rn = the Reynolds number of the ship at the input speeds

The residuary resistance coefficient (includes all resistance components other than frictional) is then calculated for each input speed as:

$$C_R = C_T - C_\text{f} \tag{6.26}$$

A new list of C_f values is computed using Reynolds numbers based on the scaled ship length, rather than the input length. A new table of C_T values is then constructed using the scaled values of C_f and the C_R values from Equation (6.26).

6.3.4 Time Domain Calculations

At each time step, C_T is interpolated. The velocity used in the interpolation is given by:

$$V_{INT} = U_G \tag{6.27}$$

where:

V_{INT} = the speed to use for the interpolation of C_T
U_G = the axial velocity of the ship (in the ship-fixed x direction)

The resistance at each time step is then computed by

$$F_{resist} = -0.5 C_{T\_INT} \rho S_I V_{INT}^2 \tag{6.28}$$

where:

F_{resist} = the resistance force computed for the time step
C_{T_INT} = the total resistance coefficient interpolated using V_{INT} from Equation (6.27)
ρ = water density
S_I = the instantaneous wetted surface obtained from an integration of panel areas
V_{INT} = the instantaneous speed as defined in Equation (6.27)

6.3.5 Reference Frame

The resistance (F_{resist}) acts in the longitudinal direction of the ship-fixed reference frame and is applied at the origin of the ship-fixed frame, so that no pitch moment is induced directly by the resistance force. The sinkage force and trim moment acting on the ship in calm water due to its forward speed can be added in separately, but will not be computed using the resistance force vector. The steady sinkage force and trim moment will not be included in the Level-0 implementation but may be introduced in Level-II or Level-III. In that case the steady sinkage force and trim moment will be determined by interpolation of a set of user-supplied force and moment coefficients similar to the coefficients C_T described in the input section. The variable V_{INT} will be used for the interpolation of these coefficients as well, but scale effects will be ignored for the sinkage force and trim moment coefficients.

6.3.6 Extension to Level-III Theory Framework

For the case where the wave making drag is captured by the slender body theory, the proposed method could be used with some minor alterations. The user could still input the total resistance curve. Some preliminary simulation runs are made at constant forward speeds and the wave resistance (R_W) is computed followed by C_W. A residuary resistance coefficient (C_R) excluding wave making drag is then computed as

$$C_R = C_T - C_f - C_W \tag{6.29}$$

C_f is then treated as described in the previous section of this paper and C_R is assumed constant with scale. A resistance coefficient that shall be called the non-wave-making resistance coefficient (C_{NW}) is computed as:

$$C_{NW} = C_R + C_f \tag{6.30}$$

And the resistance force is computed as

$$F_{resist} = -0.5 C_{NW} \rho S_I V_{INT}^2 \tag{6.31}$$

6.4 Bilge Keel forces

6.4.1 Preface

The Level-0 Tempest theory will use the Ikeda-Himeno-Tanaka (IHT) method for computing the roll moment from bilge keel damping (see Himeno (1981)). The Level-0 Tempest will not include all of the components of roll damping described in Himeno (1981), but will only include the normal force bilge keel damping force and the hull pressure bilge keel damping force. The wave making damping from the bilge keels is not included in this model. Wave making damping from the bilge keels is typically very small and can be ignored in the Level-0 Theory. However, if the bilge keel geometry is included in the added mass and damping coefficient calculations used for the radiation force model, the wave making damping from the bilge keels will be captured with no double-counting of the damping terms computed in this section.

The description of the method in Sections 6.4.2 through 6.4.6 is derived directly from a MARIN document describing work they performed for the US Navy to develop a time domain implementation of the IHT roll damping. The original IHT method was developed for frequency domain methods. There are several terms used in their discussion that are not defined in their documentation. These will be defined here, and a brief description will also be given of the user supplied input needed to implement the model. This document describes only the bilge keel damping portion of the IHT roll damping method. There are also components of roll damping from the hull frictional, eddy making and lift damping. Only the bilge keel damping will be included in the Level-0 theory for Tempest.

Definition of terms not defined in the MARIN documentation:

- $H_0(x)$ is the ratio of the local half beam to the local draft, $H_0(x) = B(x)/(2\,T(x))$
- $B(x)$ in the definition above is the local Beam at x
- the sectional area coefficient σ is the sectional area / $(B(x)\cdot T(x))$

User supplied input

The following terms will be specified at a series of stations along the length of the bilge keel (including the forward and aft ends of the bilge keels):

$B(x)$ – local waterline beam at x in meters

$T(x)$ – local sectional draft at x in meters

$S(x)$ – sectional area of the hull section at x in square meters

$b_{BK}(x)$ – bilge keel height at x in meters

$r_{BK}(x)$ – distance from the center of gravity to the bilge keel in meters at x

From the input variables listed above, the following two terms can be computed at each position along the bilge keel:

$$H_0(x) = \frac{B(x)}{2T(x)}, \qquad \sigma(x) = \frac{S(x)}{B(x)T(x)}$$

In addition to the input variables specified at positions along the length of the bilge keel, the following global input variables will be supplied.

ω_n – natural roll frequency in radians/second.

L_{BK} – overall length of the bilge keel

In equations (6.50) and (6.54), where the roll amplitude (A_ϕ) is required, a value of 10 degrees should be used.

The symbols listed in the table in the Nomenclature section apply to both the FDS and IHT models. There are several symbols listed in that table that are needed only for the FDS model. The description of the FDS model has been removed from this document, as it is based on proprietary MARIN experimental data.

6.4.2 Introduction

The (viscous) bilge keel damping is defined as the increment of roll damping when bilge keels are installed. It includes not only the roll damping of the bilge keels themselves but also all the interaction effects among the bilge keels, the hull and the waves. In the IHT-formulation, the total bilge keel damping is defined as the sum of the normal force bilge keel damping and the hull pressure bilge keel damping:

$$M_{BK} = M_{BK,N} + M_{BK,H} \tag{6.32}$$

6.4.3 IHT - Normal force bilge keel damping

The normal force bilge keel damping coefficient is first calculated for each section and then integrated along the ship length, i.e. from the bilge keel aft end to the bilge keel fore end:

$$B_{BK,N} = \int_{x_{BK,aft}}^{x_{BK,fore}} B_{BK,N}(x) \cdot dx \tag{6.33}$$

The sectional normal force bilge keel damping coefficient consists of a zero speed part and a forward speed part:

$$B_{BK,N}(x) = B_{BK,N,0}(x) + B_{BK,N,U}(x) \tag{6.34}$$

The zero speed sectional normal force bilge keel damping coefficient is expressed as

$$B_{BK,N,0}(x) = \frac{8}{3 \cdot \pi} \cdot \rho \cdot r_{BK}^2(x) \cdot b_{BK}^2(x) \cdot f^2(x) \cdot A_\phi \cdot \omega_n \cdot \left(\frac{22.5}{\pi \cdot f(x)} + \frac{r_{BK}(x) \cdot A_\phi}{b_{BK}(x)} \right) \tag{6.35}$$

where $f(x)$ is defined and elaborated on in the appendix (for layout reasons).

The forward speed sectional normal force bilge keel damping coefficient is expressed as

$$B_{BK,N,U}(x) = \frac{\pi}{2} \cdot \rho \cdot b_{BK}^2(x) \cdot r_{BK}^2(x) \cdot U \tag{6.36}$$

From the above expressions, we observe that part of the zero speed sectional normal force bilge keel damping coefficient, $B_{BK,N,0}(x)$, is proportional to the roll velocity; the other part depends on the roll amplitude only; the forward speed sectional normal force bilge keel damping coefficient, $B_{BK,N,U}(x)$, is proportional to the forward speed and does not depend on the roll velocity.

These observations allow us to write in a more direct manner

$$B_{BK,N}(U,p) = \alpha_{BK,N,0} \cdot p + \beta_{BK,N,0}(A_\phi) + \alpha_{BK,N,U} \cdot U \tag{6.37}$$

where the coefficients are given by

$$\alpha_{BK,N,0} = 2.4 \cdot \frac{8}{3 \cdot \pi} \cdot \rho \cdot \int_{x_{BK,aft}}^{x_{BK,fore}} b_{BK}(x) \cdot r_{BK}^3(x) \cdot f^2(x) \cdot dx \tag{6.38}$$

$$\beta_{BK,N,U} = \frac{22.5}{\pi} \cdot \frac{8}{3 \cdot \pi} \cdot \rho \cdot \int_{x_{BK,aft}}^{x_{BK,fore}} b_{BK}^2(x) \cdot r_{BK}^2(x) \cdot f(x) \cdot dx \tag{6.39}$$

$$\alpha_{BK,N,U} = \frac{\pi}{2} \cdot \rho \cdot \int_{x_{BK,aft}}^{x_{BK,fore}} b_{BK}^2(x) \cdot r_{BK}^2(x) \cdot dx \tag{6.40}$$

6.4.4 IHT - Hull pressure bilge keel damping

The hull pressure bilge keel damping coefficient is first calculated for each section and then integrated along the ship length, i.e. from the bilge keel aft end to the bilge keel fore end:

$$B_{BK,H} = \int_{x_{BK,aft}}^{x_{BK,fore}} B_{BK,N}(x) \cdot dx \tag{6.41}$$

The sectional hull pressure bilge keel damping coefficient is expressed as

$$B_{BK,H}(x) = \frac{4}{3 \cdot \pi} \cdot \rho \cdot r_{BK}^2(x) \cdot T^2(x) \cdot I(x) \cdot f^2(x) \cdot \omega_n \cdot A_\phi \tag{6.42}$$

From this expression we observe that $B_{BK,H,0}(x)$ is proportional to the roll velocity; therefore, we may write in a more direct manner

$$B_{BK,H} = \alpha_{BK,H} \cdot p \tag{6.43}$$

where the linear coefficient $\alpha_{BK,H}$ depends on the bilge keel parameters only:

$$\alpha_{BK,H} = \frac{4}{3 \cdot \pi} \cdot \rho \cdot \int_{x_{BK,aft}}^{x_{BK,fore}} r_{BK}^2(x) \cdot T^2(x) \cdot I(x) \cdot f^2(x) \cdot dx \tag{6.44}$$

6.4.5 IHT - Bilge keel damping – Summary (new approach)

In the new approach the bilge keel damping is calculated as

$$M_{BK}(U,p) = -\left(\alpha_{BK,N,0}\cdot|p| + \beta_{BK,N,0} + \alpha_{BK,N,U}\cdot U + \alpha_{BK,H}\cdot|p|\right)\cdot p \tag{6.45}$$

where (6.38), (6.39), (6.40) and (6.44) are used. The bilge keel damping force includes only a roll moment term. Therefore, the bilge keel force vector can be expressed as:

$$\vec{F}_{BK} = (0,0,0,M_{BK},0,0) \tag{6.46}$$

6.4.6 Calculation of underlying terms in IHT-method

$$f(x) = 1 + 0.3\cdot\exp\left(-160\cdot(1-\sigma(x))\right) \tag{6.47}$$

$$I(x) = -A_1(x)\cdot C_p^-(x) + A_2(x)\cdot C_p^+(x) \tag{6.48}$$

$$C_p^+(x) = 1.2 \tag{6.49}$$

$$C_p^-(x) = -22.5\cdot\frac{b_{BK}(x)}{\pi\cdot r_{BK}(x)\cdot f(x)\cdot A_\phi} - 1.2 \tag{6.50}$$

$$C_D(x) = C_p^+(x) - C_p^-(x) \tag{6.51}$$

$$A_1(x) = \left(m_3(x) + m_4(x)\right)\cdot m_8(x) - m_7^2(x) \tag{6.52}$$

$$A_2(x) = \frac{m_4^3(x)}{3\cdot\left(H_0(x) - 0.215\cdot m_1(x)\right)} + \frac{\left(1-m_1(x)\right)^2\cdot\left(2m_3(x) - m_2(x)\right)}{6\left(1-0.215\cdot m_1(x)\right)} + m_1(x)\cdot\left(m_3(x)\cdot m_5(x) + m_4(x)\cdot m_6(x)\right) \tag{6.53}$$

$$s_0(x) = 0.3\cdot\pi\cdot f(x)\cdot r_{BK}(x)\cdot A_\phi + 1.95\cdot b_{BK}(x) \tag{6.54}$$

$$m_1(x) = \frac{R(x)}{T(x)} \tag{6.55}$$

$$m_2(x) = \frac{OG}{T(x)} \tag{6.56}$$

$$m_3(x) = 1 - m_1(x) - m_2(x) \tag{6.57}$$

$$m_4(x) = H_0(x) - m_1(x) \tag{6.58}$$

(6.59)

$$m_6(x) = \frac{0.414 \cdot H_0(x) + 0.0651 \cdot m_1^2(x) - \left(0.382 + 0.0106 \cdot H_0(x)\right) \cdot m_1(x)}{\left(H_0(x) - 0.215 \cdot m_1(x)\right) \cdot \left(1 - 0.215 \cdot m_1(x)\right)} \tag{6.60}$$

$$m_7(x) = \begin{cases} \dfrac{s_0(x)}{T(x)} - \dfrac{\pi}{4} \cdot m_1(x) & for \quad s_0(x) > \dfrac{\pi}{4} \cdot R(x) \\ 0 & for \quad s_0(x) \le \dfrac{\pi}{4} \cdot R(x) \end{cases} \tag{6.61}$$

$$m_8(x) = \begin{cases} m_7(x) + 0.414 \cdot m_1(x) & for \quad s_0(x) > \dfrac{\pi}{4} \cdot R(x) \\ m_7(x) + \sqrt{2} \cdot \left(1 - \cos\left(\dfrac{s_0(x)}{R(x)}\right)\right) \cdot m_1(x) & for \quad s_0(x) \le \dfrac{\pi}{4} \cdot R(x) \end{cases} \tag{6.62}$$

The bilge keel radius $R(x)$ and the distance $r_{BK}(x)$ from the roll axis to the bilge keel are calculated as follows:

$$R^*(x) = 2 \cdot T(x) \cdot \sqrt{\frac{H_0(x) \cdot \left(1 - \sigma(x)\right)}{4 - \pi}} \tag{6.63}$$

$$R(x) = \begin{cases} \dfrac{1}{2} \cdot B(x) & \text{for} \quad H_0(x) \le 1 \quad \text{and} \quad \dfrac{R^*(x)}{T(x)} \ge H_0(x) \\ T(x) & \text{for} \quad H_0(x) \ge 1 \quad \text{and} \quad \dfrac{R^*(x)}{T(x)} \ge 1 \\ R^*(x) & \text{otherwise} \end{cases} \tag{6.64}$$

$$r_{BK}(x) = T(x) \cdot \left[\left\{ H_0(x) - \left(1 - \frac{1}{2}\sqrt{2}\right) \cdot \frac{R(x)}{T(x)} \right\}^2 + \left\{ 1 - \frac{OG}{\overline{T}} - \left(1 - \frac{1}{2}\sqrt{2}\right) \cdot \frac{R(x)}{T(x)} \right\}^2 \right]^{1/2} \tag{6.65}$$

6.5 Bare Hull Maneuvering forces

6.5.1 Summary of Theory

The maneuvering module in the Level-0 Tempest code will follow the ship specific maneuvering model method that is based on an Abkowitz-type, coefficient-based, maneuvering model (Abkowitz, 1962). The method assumes that data is available from rotating arm tests or PMM tests. The empirical coefficients used by the method are computed using the MANSIM program described by Kopp (2000) and Kopp (2007). An example of this maneuvering model for a containership is described in Son and Nomoto (1981, 1982) which includes all of the empirical coefficients needed to implement the model for that ship.

In many cases model test data from either rotating arm tests or PMM tests include rudder terms, while Tempest has a separate rudder model to compute the forces on the rudder. In these cases, the rudder effects are embedded in all of the measured forces and moments, not just the specific rudder angle, δ, terms. Therefore, the model test rudder terms are included in the hull forces calculations. To eliminate double counting of the rudder force, the entire calm water "rudder" force is calculated using the procedure outlined in Section 0 (with the wave orbital velocities set to zero) where these forces are then subtracted from the hull maneuvering forces. Similarly, if the empirical data is based on model tests that included a propeller, the propeller side forces will be embedded in the measured forces and moments. If this is the case, the calm water propeller side force should be subtracted from the hull maneuvering forces using the same procedure as was used for the rudder forces: computing the propeller side force using the Tempest propeller model with the wave orbital velocities set to zero and then subtracting this from the hull maneuvering force. The model test data may also include the skeg forces in all the measured forces. In the Level-0 theory, the skeg force will be left in the maneuvering force and no separate skeg force model will be applied. In some instances the model tests are performed with a "bare hull" model and therefore do not include rudder forces or propeller side forces. In these instances the rudder and propeller side forces should not be subtracted from the hull maneuvering forces, as no double counting of the rudder force is present. The input for the maneuvering model must include a flag to specify whether or not the rudder and propeller side forces are embedded in the hull maneuvering force coefficients.

This method includes the option to include a wide range of maneuvering coefficients. In most cases only a handful of the coefficients will be used. Depending on how the regression analysis is performed, different coefficients will be used or not used in the maneuvering model. The Son and Nomoto containership maneuvering model for instance does not include any odd quadratic terms (i.e. coefficients of $v|v|$), so these coefficients will all be set to zero for the containership. Son and Nomoto use only even quadratic terms and the odd terms are either linear or cubic. Other maneuvering models may include both odd quadratic and cubic terms in the equations. Several terms included in the Son and Nomoto maneuvering equations will not be implemented as they are already computed elsewhere within Tempest. The terms that will be left out are the hull resistance and propeller thrust terms in the surge equation, as well as the rudder terms in all of the equations. The Son and Nomoto model appears to compute the forces on the rudder in a manner similar to the Tempest rudder model, so these terms are simply left out. Since the rudder terms are not embedded in the coefficients, there is no need to subtract out the Tempest calm water rudder force from the maneuvering forces when the Son and Nomoto maneuvering formulation is used. Son and Nomoto also include terms proportional to roll velocity in the roll, sway, and yaw equations. It is assumed that these terms will be included within the roll damping model, so they are removed from the maneuvering equations.

6.5.2 Input

The user will supply the set of coefficients used to compute the maneuvering forces. Each coefficient will be specified at several Froude numbers. At each time step the program will compute the maneuvering forces corresponding to the Froude numbers at which the coefficients were specified. The code will then interpolate to obtain forces and moments for the Froude number based on the instantaneous speed of the ship at each time step. For the Son and Nomoto (1981) containership maneuvering model the coefficients are independent of Froude number, so no interpolation is required.

The list of coefficients which may be used in the equations for the surge, sway, roll, and yaw equations are listed in Table 6-4. Typically only ten to twelve terms in each column will be used for a given model and the rest will be set to zero.

For the heave and pitch equations, a strip-wise cross-flow drag model is used which requires the following input at set of stations along the hull:

$Cff(x), B(x), x$ specified for nx longitudinal positions along the hull

Cff is a 2-D cross flow drag coefficient used to compute the viscous damping from a 2-D hull section as a result of the heave velocity of the section. The default value will be 1.0. $B(x)$ is the local beam of the ship on the waterline. Both $Cff(x)$ and $B(x)$ will be specified at a series of longitudinal positions along the length of the hull. The drag on the 2-D section with a constant heave velocity is estimated as $Cff(x) \cdot 0.5 \cdot \rho \cdot (w(x))^2 \cdot B(x)$, where x is specified in the ship-fixed coordinate system with the origin at the center of gravity.

The coefficients for the surge, sway, roll, and yaw equations are derived from the rotating arm test data. The rotating arm test data covers a range of parameters up to a moderate heel angle, yaw rate, and sway velocity. Extrapolation outside of this range can lead to errors in the computed maneuvering forces. The user will supply maximum values of heel angle, sway velocity and yaw rate to be used in the maneuvering equations: ϕ_{max}, v_{max}, r_{max}. The absolute value of the roll angle, sway velocity, and yaw will be capped at the user specified maximum values. In most cases the rotating arm data will reference the forces and moments to the ship center of gravity. The input must specify the origin used for the empirical data. The coefficients listed in Table 6-4 should be specified in a coordinate system with x forwards, y to port, and z upwards. If the origin used to obtain the empirical coefficients is different from the center of gravity of the ship in the Tempest simulation, Tempest will first compute u, v, etc at the origin used for the empirical data. Then it will compute forces and moments at the origin used for the empirical data. These forces and moments will then be transformed to the Tempest origin, which in Level-0 is located at the center of gravity of the ship.

6.5.3 Implementation

The following terms are computed or defined outside of the maneuvering force module at each time step. They are used as input to the maneuvering force calculations:

q	Angular velocity about the ship-fixed y-axis in radians
r	Angular velocities about the ship-fixed z-axis in radians

u,v,w	Velocity of the ship at the ship-fixed reference frame origin in the ship-fixed x, y and z directions respectively.
δ	The rudder angle in radians
ρ	Water density
ϕ	Roll angle in radians

The maneuvering equations for surge, sway, roll and yaw are all written in non-dimensional form. The forces, moments, velocities and rates that appear in the equations are all in non-dimensional form. The forces and moments are non-dimensionalized using length-squared or -cubed. To account for the change in the draft of the ship as it maneuvers in waves, the ratio of the instantaneous midship draft to calm water midship draft is incorporated.

forces: $0.5 \cdot \rho \cdot L^2 \cdot U^2 \cdot Tm$;

moments: $0.5 \cdot \rho \cdot L^3 \cdot U^2 \cdot Tm$, where

ρ = density of water kg/m^3

L = length between perpendiculars

U = ship velocity, $\sqrt{u^2 + v^2}$

Tm = ratio of the instantaneous draft over the still water draft measured at midships. Tm is an attempt to scale the forces and moments by the submerged surface area.

The maneuvering equations for surge, sway, roll, and yaw are based on rotating arm test data which covers a limited range of roll angles, sway velocities, and yaw rates. For values above the maximum values specified by the user, the maximum values from the test matrix will be used in the maneuvering equations.

The non-dimensional variables for the sway velocity and yaw rate are defined as:

$v' = v/U$ (absolute value not to exceed v_{max}/U)

$r' = rL/U$ (absolute value not to exceed $r_{max}L/U$)

The roll angle, ϕ, and the rudder angle, δ, are dimensional and specified in radians. If the absolute value of the roll angle exceeds ϕ_{max}, a roll angle with magnitude ϕ_{max} and the proper sign is used in the maneuvering equations. Note that here the prime variables v' and r', are non-dimensional, whereas the variables, v and r, are dimensional. This differs from the convention used in the other sections of the theory manual, which use only dimensional velocities.

The equations used in Tempest to compute the maneuvering contribution to the non-dimensional surge and sway force and the non-dimensional roll and yaw moments are formulated using the terms listed in Table 6-4. For each equation, each coefficient is multiplied by the values listed in the last column of Table 6-4, and then the sum of all the terms gives the total maneuvering force or moment. For a given equation, many of the coefficients will be zero. The coefficients that are specified as non-zero will depend on the maneuvering model specified. Certain terms may always be zero for a symmetric ship (such as odd terms of v and r for the surge equation), but these terms are still included in Table 6-4 for completeness.

Table 6-4 Coefficients used by the Maneuvering Force Model for the Surge and Sway Force and the Roll and Yaw Moment.

Surge Coefficient	Sway Coefficient	Roll Coefficient	Yaw Coefficient	Multiplier
SurInt	SwyInt	RolInt	YawInt	1
SurR	SwyR	RolR	YawR	r'
SurRAR	SwyRAR	RolRAR	YawRAR	$r' \lvert r' \rvert$
SurR2	SwyR2	RolR2	YawR2	$(r')^2$
SurR3	SwyR3	RolR3	YawR3	$(r')^3$
SurV	SwyV	RolV	YawV	v'
SurVAV	SwyVAV	RolVAV	YawVAV	$v' \lvert v' \rvert$
SurV2	SwyV2	RolV2	YawV2	$(v')^2$
SurV3	SwyV3	RolV3	YawV3	$(v')^3$
SurP	SwyP	RolP	YawP	ϕ
SurPAP	SwyPAP	RolPAP	YawPAP	$\phi \lvert \phi \rvert$
SurP2	SwyP2	RolP2	YawP2	ϕ^2
SurP3	SwyP3	RolP3	YawP3	ϕ^3
SurD	SwyD	RolD	YawD	δ
SurD2	SwyD2	RolD2	YawD2	δ^2
SurD3	SwyD3	RolD3	YawD3	δ^3
SurRV	SwyRV	RolRV	YawRV	$r' v'$
SurV2R	SwyV2R	RolV2R	YawV2R	$r' (v')^2$
SurR2V	SwyR2V	RolR2V	YawR2V	$(r')^2 v'$
SurV2R2	SwyV2R2	RolV2R2	YawV2R2	$(r')^2 v')^2$
SurRAV	SwyRAV	RolRAV	YawRAV	$r' \lvert v' \rvert$
SurVAR	SwyVAR	RolVAR	YawVAR	$\lvert r' \rvert v'$
SurPAR	SwyPAR	RolPAR	YawPAR	$\phi \lvert r' \rvert$
SurRAP	SwyRAP	RolRAP	YawRAP	$\lvert \phi \rvert r'$
SurPR2	SwyPR2	RolPR2	YawPR2	$\phi (r')^2$
SurRP2	SwyRP2	RolRP2	YawRP2	$\phi^2 r'$
SurPAV	SwyPAV	RolPAV	YawPAV	$\phi \lvert v' \rvert$
SurVAP	SwyVAP	RolVAP	YawVAP	$\lvert \phi \rvert v'$
SurPV2	SwyPV2	RolPV2	YawPV2	$\phi (v')^2$
SurVP2	SwyVP2	RolVP2	YawVP2	$\phi^2 v'$
SurRD	SwyRD	RolRD	YawRD	$r' \delta$
SurVD	SwyVD	RolVD	YawVD	$v' \delta$
SurDR2	SwyDR2	RolDR2	YawDR2	$(r')^2 \delta$
SurDV2	SwyDV2	RolDV2	YawDV2	$(v')^2 \delta$
SurRD2	SwyRD2	RolRD2	YawRD2	$r' \delta^2$
SurVD2	SwyVD2	RolVD2	YawVD2	$v' \delta^2$
SurDAR	SwyDAR	RolDAR	YawDAR	$\lvert r' \rvert \delta$
SurDAV	SwyDAV	RolDAV	YawDAV	$\lvert v' \rvert \delta$
SurRAD	SwyRAD	RolRAD	YawRAD	$r' \lvert \delta \rvert$
SurVAD	SwyVAD	RolVAD	YawVAD	$v' \lvert \delta \rvert$

$$F_x' = \text{SurInt} + r'\cdot\text{SurR} + r'|r'|\cdot\text{SurRAR} + (r')^2\cdot\text{SurR2} + \ldots$$
$$F_y' = \text{SwyInt} + r'\cdot\text{SwyR} + r'|r'|\cdot\text{SwyRAR} + (r')^2\cdot\text{SwyR2} + \ldots \quad (6.66)$$
$$M_x' = \text{RolInt} + r'\cdot\text{RolR} + r'|r'|\cdot\text{RolRAR} + (r')^2\cdot\text{RolR2} + \ldots$$
$$M_z' = \text{YawInt} + r'\cdot\text{YawR} + r'|r'|\cdot\text{YawRAR} + (r')^2\cdot\text{YawR2} + \ldots$$

The forces will then be converted to dimensional form (in N and N-m) and added to the other force components.

$$F_x = F_x'\cdot 0.5\cdot\rho\cdot L^2\cdot U^2\cdot Tm \quad , \quad F_y = F_y'\cdot 0.5\cdot\rho\cdot L^2\cdot U^2\cdot Tm \quad (6.67)$$
$$M_x = M_x'\cdot 0.5\cdot\rho\cdot L^3\cdot U^2\cdot Tm \quad , \quad M_z = M_z'\cdot 0.5\cdot\rho\cdot L^3\cdot U^2\cdot Tm$$

F_x, F_y, M_x and M_z are already defined in the ship-fixed reference frame and are added to the other force components to compute the total force and moment acting on the ship at each time step.

The heave and pitch maneuvering forces are computed by computing the cross flow drag at a set of stations along the length of the hull, and then integrating over the length of the hull to compute the heave force and pitch moment. The local heave velocity of a section located at longitudinal position x from the center of gravity can be computed from the heave velocity at the center of gravity, w, and the rotational velocity about the ship-fixed y-axis, q:

$$w(x) = w - q\cdot x \quad (6.68)$$

The vertical cross flow drag force on the section is then computed as:

$$Drag = \frac{1}{2}\rho\cdot Cff(x)\cdot B(x)\cdot\left(w(x)\right)^2 \quad (6.69)$$

The heave force, F_z, and pitch moment, M_y, are then computed by integrating the drag over the length of the hull:

$$F_z = \frac{1}{2}\rho\int Cff(x)\cdot B(x)\cdot\left(w(x)\right)^2 dx \quad (6.70)$$

$$M_y = \frac{1}{2}\rho\int x\cdot Cff(x)\cdot B(x)\cdot\left(w(x)\right)^2 dx \quad (6.71)$$

The integration is performed over the wetted length of the hull. A trapezoidal rule integration scheme can be used. The heave force and pitch moment computed using the above formulae are in dimensional form and are defined in the ship-fixed frame. F_z and M_y are added to the other force components to compute the total force and moment action on the ship at each time step. The complete bare hull maneuvering force vector is then defined as:

$$\vec{F}_{man} = (F_x, F_y, F_z, M_x, M_y, M_z) \quad (6.72)$$

6.5.4 Zero Speed Performance

Due to the fact that these terms are divided by ship speed, non-dimensional sway velocity, v', and yaw rate, r', have the potential to be very large as ship speed approaches zero. This is avoided by treating some terms differently as ship speed approaches zero. The uncertainty of the maneuvering model coefficients at these speeds warrants this pragmatic approach.

Speeds less than 0.005m/s are considered zero speed. In this case, Fn, non-dimensional sway velocity and yaw rate are all set to zero in the calculations. This avoids division by zero errors when non-dimensionalizing sway velocity and yaw rate.

As the non-dimensional forces and moments are multiplied by velocity squared, terms less than third order, e.g., $r'v'$ or $(v')^2$, are well behaved as ship speed goes to zero. The coefficients for third order and higher terms, e.g., $(r')^2v'$, are linearly reduced to zero for $Fn < 0.015$.

The fraction of Tempest rudder forces subtracted from the hull forces is linearly reduced from 100% to 0% for $Fn < 0.015$. While this appears to be double counting the rudder at near zero speeds, it is actually gradually switching rudder models at near zero speeds. Tests indicated the coefficient based rudder model acted as though there was no rudder at near-zero speeds.

6.6 Rudder and Fin forces

6.6.1 Summary of Theory

This section describes the theory for computing the rudder force for a ship in a seaway. The method will be implemented in the time domain dynamic stability code Tempest. The theory is based primarily on the empirical formulae developed by Wicker and Fehlner (1958) and is developed for the Level-0 implementation of Tempest.

With some refinements, the theory may also be appropriate for the higher levels of Tempest. A brief synopsis of the theory will be described in this section, with the details provided in the implementation section below. The ships for which Tempest is currently envisioned all have moveable rudders, as opposed to flapped rudders, semi-balanced rudders on a horn, etc. Therefore the rudder model described here applies only to all movable rudders and fins. Prior to the start of the time stepping calculations the program will establish formulae for the lift and drag coefficients of the rudder as a function of angle of attack based on empirical data. Then at each time step the local flow velocity at the rudder will be computed. The velocity will include the inflow resulting from the motion of the ship, the wave orbital velocity, and the propeller slipstream. The rudder deflection angle will be provided at each time step from the autopilot algorithm, which is described in a separate document. From the local velocity vector and the rudder deflection angle, the angle of attack of the rudder will be determined, which will be used to compute the lift and drag on the rudder. The lift and drag forces will be transformed to forces and moments acting on the ship in the ship-fixed reference frame. If the rudder is only partially submerged, the forces will be adjusted by the ratio of the submerged area of the rudder to the total area of the rudder.

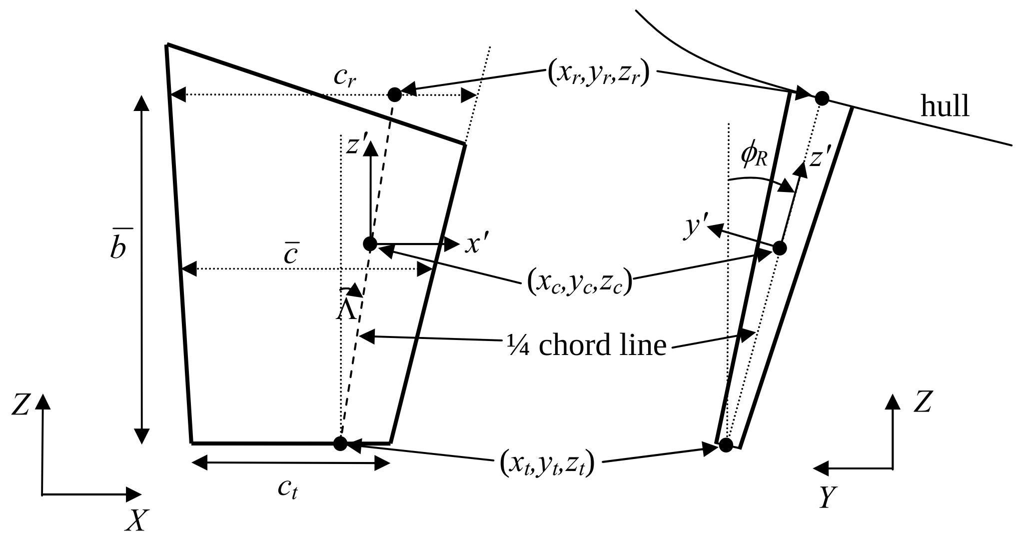

Figure 6-1 Definition of some rudder parameters. (Y axis into page on left half figure, X axis into page on right half figure. The value of ϕ_R is negative as shown)

6.6.2 Input

The user will be required to provide the following input for the rudder. The first set of input is required to establish the position and geometry of the rudder and the second set of input defines some of the coefficients required for computing the rudder force.

The position and geometry of the rudder will be computed from user specified values for the chord length of the rudder at the root and tip, and the (x,y,z) offsets of the quarter-chord point at the root and tip of the rudder: c_r, c_t, (x_r,y_r,z_r) and (x_t,y_t,z_t) shown in Figure 6-1.

The other geometric rudder parameters such as area, span, dihedral angle, etc. can all be computed from these values. Internally the offsets of the quarter-chord point at the tip (x_t,y_t,z_t) and at the root (x_r,y_r,z_r) will be referenced to the ship-fixed origin. The user will enter the distances to these points forward from the AP and above the baseline. This will be converted internally to the position of the quarter-chord points relative to the ship CG in the ship-fixed frame.

The user must specify several other parameters to compute the lift and drag on the rudder. Some values will have preset default values, in which case the user will have the option to specify a value. The additional inputs are:

- a_e – the effective aspect ratio of the rudder. The geometric aspect ratio, a, of the rudder can be computed as: $\bar{b}/\bar{c}$. The *effective* aspect ratio accounts for the mirror image effect when the rudder is flush with the hull. Typically the effective aspect ratio is assumed to be twice the geometric aspect ratio. However, a different value may be specified if a large rudder stool is used or the rudder has a large tip plate. A large rudder stool separates the rudder from the hull which reduces a_e, while a tip plate would increase the loading near the rudder tip, which increases a_e. The effective aspect ratio is used when computing the lift and drag coefficients and the stall angle of the rudder.

- γ_R - flow straightening coefficient. This is used to adjust the angle of attack of the rudder to account for the flow straightening influence of the hull and propeller
- C_{d0} – minimum sectional drag coefficient. The default value will be 0.0065, which corresponds to a NACA 0015 section
- C_{DC} – crossflow drag coefficient from Whicker and Fehlner (1958). The user will have the option to either enter this coefficient directly, or to specify whether the tip shape is square or faired and have the program compute the coefficient based on the data in Whicker and Fehlner (1958) (which expressed the coefficient as a function of the tip shape and taper ratio).
- δ – rudder deflection angle. This will be provided by the controller algorithm described in Section 5 at each time step.
- α_S – stall angle. The program will compute the default value of the stall angle using the formula listed in "Step 1" of the implementation section. The user may choose to override the default value by specifying a value directly
- C_N – Normal force coefficient for stalled wing. The default value will be 1.8 as recommended by Hoerner (1975).

6.6.3 Implementation

Step 1 is performed before the start of the time stepping procedure. The remaining steps are performed at each time step. All the steps will be performed separately for each rudder on the ship.

Step 1. Read input, compute time independent values

Prior to the start of the time stepping calculations the program will first read in all the user supplied inputs. Then the geometry information required for computing the force and center of pressure for the rudder will be computed.

The mean chord and span are computed as:

$$\bar{c} = \frac{1}{2}(c_t + c_r) \quad , \quad \bar{b} = \sqrt{(y_t - y_r)^2 + (z_t - z_r)^2} \tag{6.73}$$

Then the area and geometric aspect ratio of the rudder can be computed:

$$A_R = \bar{b}\,\bar{c} \quad , \quad a = \bar{b}/\bar{c} \tag{6.74}$$

The dihedral angle of the rudder and the sweep angle of the quarter chord line will be computed as:

$$\phi_R = \arctan\left(\frac{(y_r - y_t)}{\bar{b}}\right) \quad , \quad \Lambda = \arctan\left(\frac{(x_r - x_t)}{\bar{b}}\right) \tag{6.75}$$

The spanwise location of the center of pressure will be computed by assuming an elliptical spanwise loading distribution, and in the chordwise direction it will be assumed that the center of pressure lies on the quarter chord line. With these assumptions the location of the center of pressure is:

$$\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix} + \frac{4}{3\pi}\begin{pmatrix} x_t - x_r \\ y_t - y_r \\ z_t - z_r \end{pmatrix} \tag{6.76}$$

The velocity will be computed at each time step at the mid-span point along the quarter chord line:

$$\begin{pmatrix} x_m \\ y_m \\ z_m \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x_t + x_r \\ y_t + y_r \\ z_t + z_r \end{pmatrix} \tag{6.77}$$

The velocity will first be computed in the ship-fixed reference frame, and then be transformed to a frame with the z axis parallel to the rudder stock, as indicated by the axis system marked (*x',y',z'*) in Figure 6-1. The transformation matrix used to compute velocities in the rudder frame from velocities defined in the ship-fixed frame is:

$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_R & -\sin\phi_R \\ 0 & \sin\phi_R & \cos\phi_R \end{bmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{6.78}$$

where (*u,v,w*) is the velocity vector in the ship-fixed reference frame and (*u',v',w'*) is the velocity in the rudder frame. The transformation matrix is orthogonal, so its inverse is equal to its transpose.

The lift slope coefficient, $dC_L/d\alpha$, and the stall angle, α_S, can be computed based on the geometry and user specified value of the effective aspect ratio. The formula used for the lift slope coefficient is that developed by Whicker and Fehlner (1958) based on their experimental data:

$$\frac{dC_L}{d\alpha} = \frac{(0.9)\,2\pi\,a_e}{\left(\cos\Lambda\sqrt{\dfrac{a_e^2}{\cos^4\Lambda}+4}\right)+1.8} \tag{6.79}$$

If a value for the cross flow drag is not specified by the user, a value should be computed based on the Whicker and Fehlner (1958) data:

$$\lambda = c_t/c_r$$
$$C_{DC} = 0.1 + 0.7\lambda \quad , \quad \text{for rudders with faired tips} \tag{6.80}$$
$$C_{DC} = 0.1 + 1.6\lambda \quad , \quad \text{for rudders with square tips}$$

Whicker and Fehlner (1958) recorded the stall angles for the fins examined in their wind tunnel experiments and Lloyd (1989) fit formulas to their data. The rudder will operate in the slipstream of the propeller, and the turbulence in this flow delays the stall to a higher angle from what was recorded in the Whicker and Fehlner (1958) wind tunnel tests. If a value for stall angle is not specified by the user, Tempest will use the formula derived by Lloyd(1989) to specify the rudder stall angle with an additional 10 degrees added to account for the turbulence in the propeller slipstream:

$$\alpha_S = 1.225 - 0.445 a_e + 0.075 a_e^2 \quad \text{radians} \quad \text{for } a_e < 3.0$$

$$\alpha_S = 0.565 \quad \text{radians} \quad \text{for } a_e > 3.0 \tag{6.81}$$

Step 2. Compute velocity vector at midspan point on the rudder.

At each time step the velocity vector at the mid-span point along the rudder quarter-chord line (x_m, y_m, z_m) is computed. The velocity is computed by first adding the velocity from the motion of the ship to the wave orbital velocity, and then adding the velocity added by the propeller slipstream. The procedure for computing the wave orbital velocity at an arbitrary point in space is described in a separate document. The computations for the wave orbital velocity are performed first in the earth-fixed frame, but must be transformed to the ship-fixed frame before being added to the velocities from the motion of the ship. The velocity vector at the rudder mid-span point (x_m, y_m, z_m) is computed as:

$$\vec{U}_R = -\begin{pmatrix} u(1-w_F) + z_m q - y_m r \\ v + x_m r - z_m p \\ w + y_m p - x_m q \end{pmatrix} + \begin{pmatrix} u_w \\ v_w \\ w_w \end{pmatrix} - \begin{pmatrix} U_{PROP} \\ 0 \\ 0 \end{pmatrix} , \tag{6.82}$$

where: (u,v,w) is the velocity of the ship defined at the ship-fixed origin, (p,q,r) is the rotational velocity of the ship, (u_w, v_w, w_w) is the wave orbital velocity at the center of the propeller disk, w_F is the wake fraction coefficient, which is discussed in Section 6.7, and U_{PROP} is the added axial velocity from the propeller. All the above quantities are defined in the ship-fixed reference frame. The negative sign before the first and last vectors in the formula for $\vec{U}_R$ indicates that the inflow velocity into the rudder is in the opposite direction from the ship motion and the additional velocity from the propeller slipstream acts along the negative x axis in the ship frame.

The additional velocity in the ship-fixed x direction from the propeller slipstream is computed following the method described in Söding (1981 and 1999). First the axial propeller slipstream velocity far downstream from the propeller is computed from the momentum equation as:

$$V_\infty = U_A \sqrt{1 + C_T} \tag{6.83}$$

From the propeller force calculations, the values for K_T and J are available, and from these C_T and U_A can be computed as:

$$C_T = \frac{T}{\rho \pi U_A^2 D^2 / 8} = \frac{8 K_T}{\pi J^2} \quad , \qquad U_A = J n D \tag{6.84}$$

The radius of the propeller slipstream far downstream from the propeller based on continuity is:

$$R_\infty = R_O \sqrt{\frac{V_\infty + U_A}{2V_\infty}} \tag{6.85}$$

where R_O is the propeller radius equal to ½D. The radius of the propeller slipstream at the location of the rudder is approximated by the formula:

$$R_{X_{RUD}} = R_O \frac{0.14\left(R_\infty/R_O\right)^3 + \left(R_\infty/R_O\right)\xi^{1.5}}{0.14\left(R_\infty/R_O\right)^3 + \xi^{1.5}} \quad , \tag{6.86}$$

where ξ is the distance from the propeller to the rudder normalized by the propeller radius:

$$\xi = \frac{(x_p - x_m)}{R_O} \tag{6.87}$$

The velocity in the propeller slipstream at the location of the rudder is then computed from the continuity equation:

$$V_{X_{RUD}} = V_\infty \frac{R_\infty^2}{R_{X_{RUD}}{}^2} \tag{6.88}$$

The additional velocity from the propeller is added to the rudder inflow velocity, which can then be computed by subtracting the inflow velocity to the propeller:

$$U_{PROP} = V_{X_{RUD}} - U_A \tag{6.89}$$

For the inclusion of the propeller slipstream velocity on the rudder, the inclination of the propeller shaft is ignored (at least for Level-0). Söding (1999) and Brix (1993) also include some additional corrections to the influence of the propeller on the rudder lift by accounting for the influence of turbulent mixing in the propeller slipstream and the influence of the finite width of the slipstream. These terms will not be included in the Level-0 implementation of Tempest.

Step 3. Compute angle of attack for the rudder.

To compute the angle of attack on the rudder, the inflow velocity into the rudder is first transformed into the rudder frame (the (*x'*,*y'*,*z'*) frame shown in Figure 6-1).

$$\vec{U}_R{}' = \begin{pmatrix} u_R' \\ v_R' \\ w_R' \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_R & -\sin\phi_R \\ 0 & \sin\phi_R & \cos\phi_R \end{bmatrix} \vec{U}_R \tag{6.90}$$

The angle of attack is then determined by examining the velocity components in the *x'y'* plane as shown in Figure 6-2. The angle of the inflow into the rudder relative to the *x'* axis is computed as:

$$\beta = \arctan\left(\frac{v'_R}{u'_R}\right) \tag{6.91}$$

This angle is adjusted by the flow straightening coefficient, γ_R, to account for the flow straightening influence of the hull and propeller. Molland and Turnock (2002) discussed the use of the flow straightening coefficient and performed a set of experiments to determine values of γ_R for several simple configurations. In Tempest, the user will provide a value of γ_R as input, with the default value being 1.0.

$$\beta' = \gamma_R \beta \tag{6.92}$$

The angle of attack is then computed by subtracting β' from the rudder deflection angle, δ, which is provided by the autopilot routine at each time step:

$$\alpha = \kappa_\delta \delta - \beta' \tag{6.93}$$

The rudder deflection angle δ is defined as positive using a right hand rule about the z' axis, which points upwards (out of the paper in Figure 6-2) resulting in a turn to starboard as shown in Figure 6-2. The positive direction for the angles α and β are defined in the same manner. The κ_δ factor accounts for fixed portions of the rudder (the rudder stool) that effectively reduce the angle of attack. κ_δ is set equal to the ratio of the planform area of the movable rudder over the total planform area of the rudder and stool. For small to moderate rudder deflection angles, the rudder and stool are modeled as a single lifting surface with a variation in angle of attack along the span.

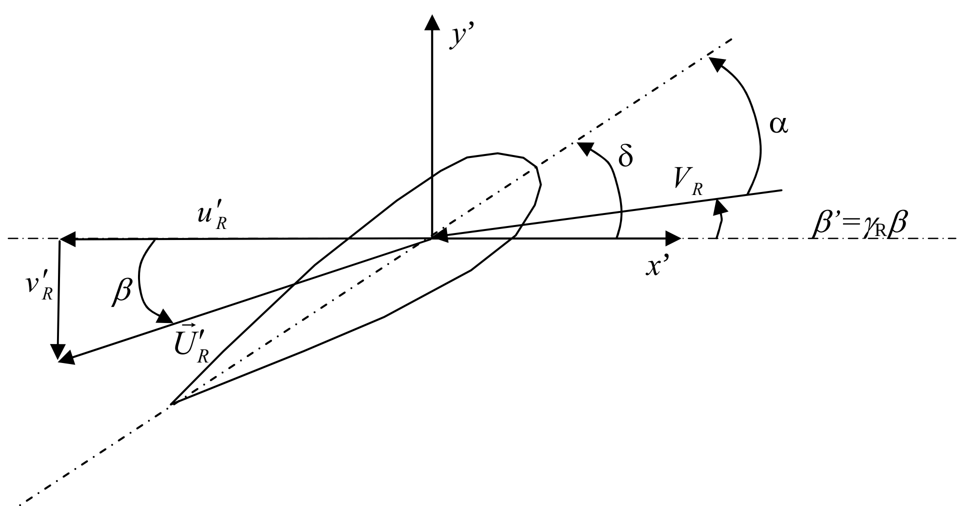

Figure 6-2 Inflow into the rudder and calculation of angle of attack.

Step 4. Compute the Lift and Drag on the rudder

For cases where the angle of attack is below the stall angle, the lift and drag coefficients are computed from the empirical formulae derived in Whicker and Fehlner (1958).

For $|\alpha| \le \alpha_S$:

$$C_L = \left(\frac{dC_L}{d\alpha}\right)\alpha + \frac{C_{DC}}{a_e}\alpha|\alpha|$$

$$C_D = C_{d0} + \frac{C_L^2}{0.9\pi a_e} \tag{6.94}$$

where $\frac{dC_L}{d\alpha}$ and C_{DC} were computed in Step 1. For cases where the angle of attack is greater than the stall angle, it will be assumed that the flow is completely separated at the leading and trailing edge of the rudder, and the force is normal to the chord line of the rudder. The resultant lift and drag coefficients in this condition are computed from the formulae found in Hoerner (1975):

For $|\alpha| > \alpha_S$,

$$C_L = B \cdot C_N \sin\alpha\cos\alpha + C_{L,\max} \cdot B_0 \tag{6.95}$$

$$C_D = B \cdot C_N \sin^2\alpha + C_{D,\max} \cdot B_0$$

where B, B_0, $C_{L,max}$, and $C_{D,max}$ have been included to create a cubic transition between the fully stalled flat plate and the maximum C_L condition when $|\alpha| > \alpha_S$ and are defined as:

for $|\alpha| > 1.25 \cdot \alpha_S$: $\quad B = 1$

for $|\alpha| \le 1.25 \cdot \alpha_S$: $\quad B = \left(3.0 \cdot u - 2 \cdot u^2\right) \cdot u$

where $\quad u = \dfrac{4.0 \cdot (|\alpha| - \alpha_S)}{\alpha_S}$

$$C_{L,\max} = \frac{dC_L}{d\alpha} \cdot \alpha_S + \frac{C_{DC}}{a_e} \cdot \alpha_S \cdot |\alpha_S| \tag{6.96}$$

$$B_0 = 1.0 - B$$

$$C_{D,\max} = C_{d0} + \frac{{C_{L,\max}}^2}{0.9 \cdot \pi \cdot a_e}$$

After the lift and drag coefficients are computed, the lift and drag can be calculated. The velocity used to compute the forces from the lift and drag coefficients is the component of the inflow velocity to the rudder in the *x'y'* plane.

$$V_R^2 = \left(u_R^{'}\right)^2 + \left(v_R^{'}\right)^2 \tag{6.97}$$

$$L = \tfrac{1}{2}\rho V_R^2 A_R C_L \tag{6.98}$$

$$d = \tfrac{1}{2}\rho V_R^2 A_R C_D$$

The lower case d is used here for drag to avoid confusion with the symbol used earlier for the propeller diameter.

Step 5. Correct rudder lift and drag for partial rudder submergence.

If the rudder is not completely submerged, the lift and drag on the rudder will be corrected to account for this. The lift and drag will be adjusted based on the approximate percentage of the rudder area that is submerged. This is equivalent to replacing A_R in the formulae for lift and drag listed above with the submerged rudder area A_{RS}. To approximate A_{RS} the wave elevation at the rudder is first computed. Let z_{WR} be the wave elevation above the rudder location (x_m, y_m). If the rudder tip is above the wave: $z_t > z_{WR}$, then the rudder is completely out of the water and the rudder lift and drag are set to zero. If the rudder root is below the wave: $z_r > z_{WR}$, then the rudder is completely submerged and no adjustment to the lift and drag from step 4 is required. Otherwise the submerged area of the rudder is computed as follows. First compute f, the percentage of the rudder span that is submerged:

$$f = \frac{z_{WR} - z_t}{z_r - z_t} \tag{6.99}$$

The submerged area of the rudder can then be approximated as:

$$A_{RS} = c_t f \,\overline{b} + \tfrac{1}{2}\left(c_r - c_t\right) f^2 \overline{b} \tag{6.100}$$

The lift and drag on the rudder are then recalculated using A_{RS} in place of A_R.

$$L = \tfrac{1}{2}\rho V_R^2 A_{RS} C_L \tag{6.101}$$

$$d = \tfrac{1}{2}\rho V_R^2 A_{RS} C_D$$

Step 6. Compute rudder forces and moments in the ship-fixed frame.

The lift and drag force on the rudder act in the $x'y'$ plane in the direction perpendicular and parallel to the inflow velocity to the rudder defined by the angle β'. To resolve the forces in the ship-fixed frame, the forces are first transformed to forces in the x' and y' directions, and then are transformed to the ship-fixed frame. The forces in the x' and y' directions are:

$$F_{XR}^{'} = -d \cos \beta' - L \sin \beta' \tag{6.102}$$

$$F_{YR}^{'} = L \cos \beta' - d \sin \beta'$$

The forces are then transformed to the ship-fixed frame using the transpose of the transformation matrix defined in Step 1.

$$\vec{F}_R = \begin{pmatrix} F_{RX} \\ F_{RY} \\ F_{RZ} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_R & \sin\phi_R \\ 0 & -\sin\phi_R & \cos\phi_R \end{bmatrix} \begin{pmatrix} F'_{RX} \\ F'_{RY} \\ 0 \end{pmatrix} \tag{6.103}$$

The rudder force acts at the center of pressure, (x_c, y_c, z_c), which was computed in Step 1. The moments which result from the rudder force are computed from $\vec{F}_R$ and (x_c, y_c, z_c):

$$\vec{M}_R = \begin{pmatrix} F_{RZ}\, y_c - F_{RY}\, z_c \\ F_{RX}\, z_c - F_{RZ}\, x_c \\ F_{RY}\, x_c - F_{RX}\, y_c \end{pmatrix} \tag{6.104}$$

$\vec{F}_R$ and $\vec{M}_R$ are the force and moment from the rudder acting at the ship-fixed reference frame origin, which is the ship CG in Level-0. These two vectors are added to the other force components to compute the total force and moment action on the ship at each time step.

6.6.4 Notes on ship specific empirical data

Rudder data either from measurements in the Large Cavitation Tunnel (LCC) or from CFD computations are available for the rudder designs for some ships. When available this data should be used to help specify the lift and drag coefficients more accurately for the rudder model implemented in Tempest. Within the Level-0 theory discussed in this document, the data from LCC tests could be used to specify the stall angle and to determine a value for the effective aspect ratio that closely matches the lift slope curve.

One difficulty in basing the lift and drag coefficients solely on the data from the LCC tests, is that these tests have been performed only with the ship traveling straight ahead with the rudder deflected at different angles. The influence of the ship drift angle and cross flow from the wave induced velocity would need to be accounted for in some manner. If the stool on which the rudder sits has a longer span than a typical rudder stool, the lift on the rudder stool should be considered. The rudder force model currently computes the force only on the rudder, not on the stool. The Level-0 maneuvering force model was developed from rotating arm test data with the rudders and rudder stool in place, and the rotating arm tests measured the total force on the model including the forces from rudder and the rudder stool. The force on the rudder is then computed and subtracted out to obtain force coefficients for building the maneuvering model. However the force from the rudder stool should still be included in the maneuvering force model, so no separate calculation of the force from the rudder stool should be required, if the Level-0 maneuvering model is used.

The stall angle, α_S, can be estimated from empirical data. The stall angle is dependent primarily on the aspect ratio, profile shape, thickness, Reynolds number, surface roughness, and the turbulence in the inflow (Brix 1993). This makes an accurate prediction of the stall angle difficult. Figure 6-3 shows the empirical formula used in Level-0 Tempest which is based on Lloyd's analysis of the Whicker and Fehlner data.

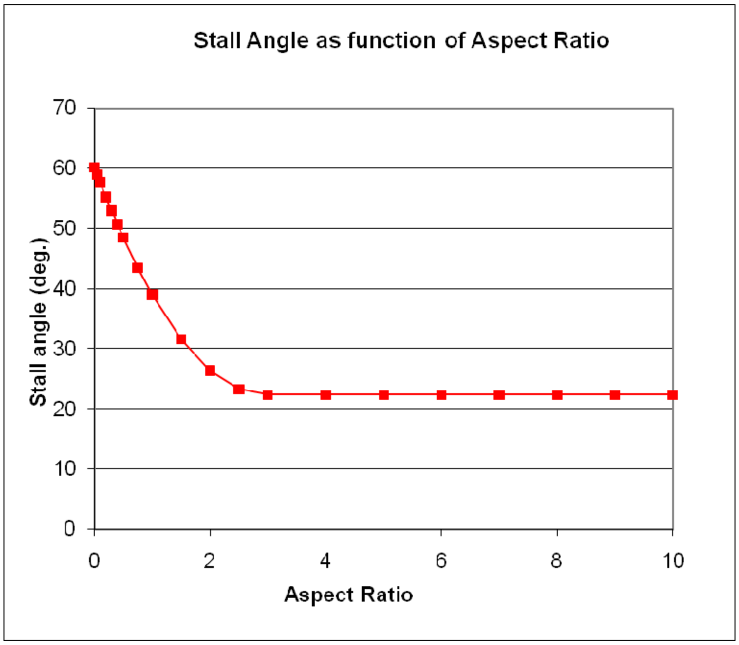

Figure 6-3 Stall angle formula derived by Lloyd (1989) based on Whicker and Fehlner (1958)

Several empirical formulae are available for computing the lift coefficient slope ($dC_L/d\alpha$), as a function of effective aspect ratio. Figure 6-4 compares the formulae proposed by Hoerner (1975), Söding (1999) and Whicker and Fehlner (1958). There is much less empirical data available for the lift and drag on rudders at angles of attack significantly over the stall angle. Figure 6-5 is taken from Hoerner (1975) and show the lift from several 2-D foil experiments. Figure 6-6 shows the computed lift and drag coefficients using the formulae specified for the Tempest Level-0 theory over the full range of angles of attack from 0° to 180° for a rectangular foil with an aspect ratio of 2. The curve for the lift coefficient compares well with the 2-D test data from Hoerner (1975).

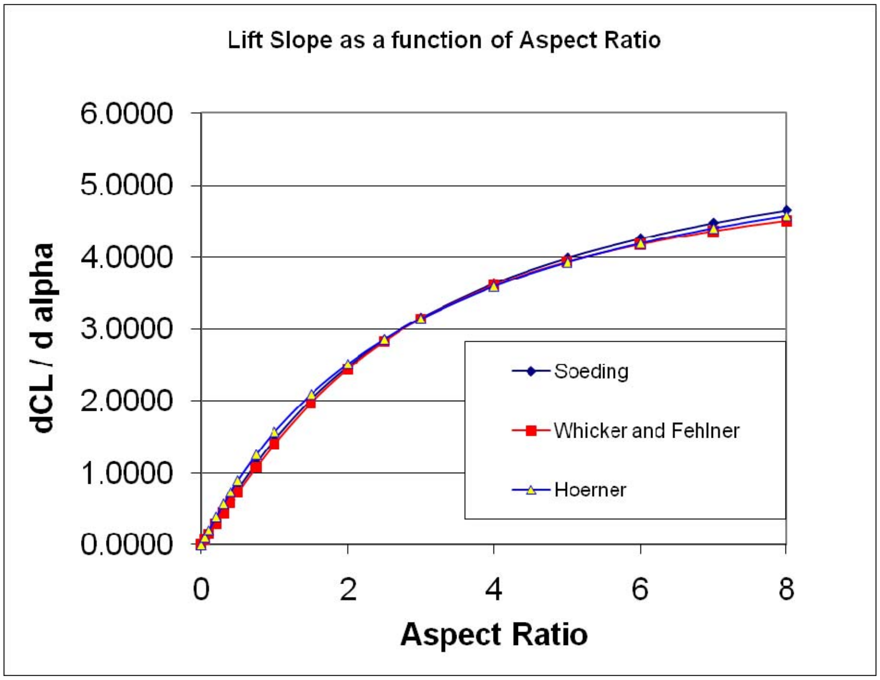

Figure 6-4 Comparison of formulae for lift coefficient slope.

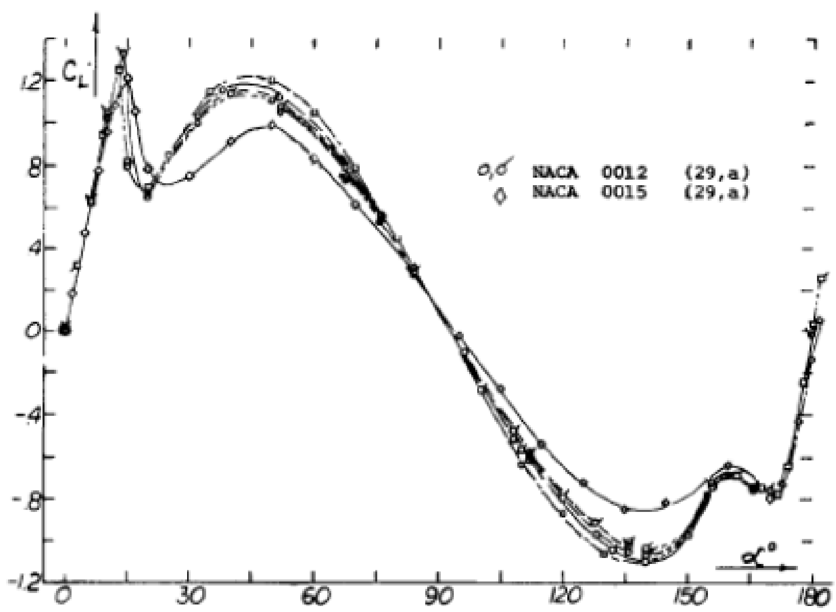

Figure 33. Variation of lift coefficient for angles of attack 0 to 180 degrees.

Figure 6-5 Lift on 2D foils, past stall angle, reproduced from Hoerner (1975)

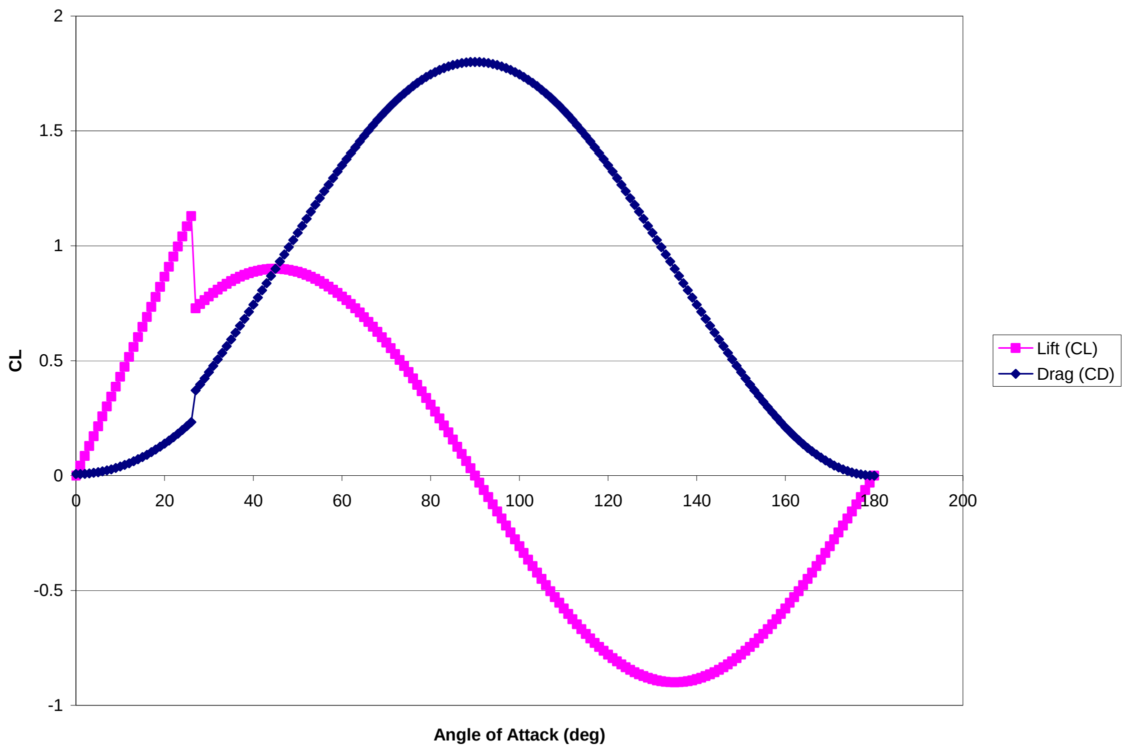

Figure 6-6 Lift and Drag coefficients computed for a rectangular foil with an aspect ratio of 2.0, based on the formulae described in the Tempest Level-0 theory.

6.7 Propulsor Force

6.7.1 Summary of Theory

This section describes the theory for computing the propeller force for a ship in a seaway. This method will be implemented in the time domain dynamic stability code Tempest. The theory is initially developed for the Level-0 implementation of Tempest.

With some refinements, the theory may also be appropriate for the higher levels of Tempest. The theory is developed initially for conventional fixed-pitch propellers. Extensions to the theory would be required to model controllable pitch propellers, waterjets, and other non-conventional propulsors.

A brief synopsis of the theory will be described in this section, with the details provided in the implementation section below. The propeller thrust will be determined by interpolating the user-supplied open water propeller thrust curve (K_T vs. J) at each time step. The advance coefficient, J, will be computed at each step from the instantaneous motion of the ship and the wave orbital velocity. The velocity will be computed at a single point located at the center of the propeller disk. The velocity vector at this point will be decomposed into a component parallel to the propeller shaft and a cross flow velocity component. The influence of the ship hull on the propeller inflow will be accounted for using the Taylor wake fraction coefficient and the increase in hull resistance from the propeller will be accounted for using a thrust deduction fraction.

Propeller forces that are perpendicular to the shaft axis resulting from cross flow will be estimated using empirical formulae described by Fuhs and Dai (2007). If the propeller is only partially submerged, the forces will be adjusted by the ratio of the submerged area of the propeller disk to the total area of the disk.

6.7.2 Input

The user will be required to provide the following input to characterize the propeller performance. The user will supply a set of points describing the open water propeller thrust curve: K_T vs. J. The coefficients K_T and J are defined below:

$$K_T = \frac{T}{\rho n^2 D^4} \tag{6.105}$$

$$J = \frac{V_A}{n D} \tag{6.106}$$

The curve should be defined by a sufficient number of points for K_T to be interpolated at each time step, and should include the entire first quadrant of the open water curve including a value for $J = 0$ up to the value of J where $K_T = 0$. Negative values of J may also be entered, to allow for reverse flow into the propeller.

In addition the user will supply the following values:

D	diameter of propeller
rpm	rotations per minute (converted to rotations per second internally)
ρ	density of water
w_F	Taylor wake fraction coefficient
t_P	thrust deduction coefficient
x_P, y_P, z_P	position of the center of the propeller disk in ship-fixed frame (The user will enter the distance of the propeller forward from the AP and above the baseline, this will be converted internally to the position of the propeller relative to the ship CG in the ship-fixed frame.)
α_S, β_S	angles defining propeller shaft orientation
γ	flow straightening factor
RHP	integer flag to indicate whether propeller is right hand turning (+1) or left hand turning (-1). A right hand turning propeller will turn clockwise when viewed from the stern looking forward.

The Taylor wake fraction coefficient is used to adjust the velocity into the propeller to account from the presence of the ship hull and is defined as:

$$w_F = \frac{V - V_A}{V} \quad , \tag{6.107}$$

where V is the axial velocity at the location of the propeller in the absence of the ship hull and V_A is the axial component of local velocity vector at the center of the propeller disk in the presence of the hull.

The thrust deduction fraction describes the additional resistance on the hull resulting from the propeller – hull interactions. Traditionally this additional resistance is thought of as a reduction of the propeller thrust. The thrust deduction fraction is defined as:

$$t_P = \frac{T - R_{hull}}{T} \quad , \tag{6.108}$$

where R_{hull} is the towed resistance of the hull without the propeller and T is the propeller thrust. Methods for determining w_F and t_P for twin screw destroyers can be found in Hurwitz (1980).

The flow straightening factor is an optional input variable that will be used to adjust the magnitude of the cross flow velocity to account for the influence the ship hull and skeg have on straightening the flow into the propeller. It is defined as:

$$\gamma = {\phi_S}'/\phi_S \quad , \tag{6.109}$$

where ϕ_S is the angle between the total velocity vector at the propeller and the shaft axis in the absence of the hull and ϕ_S' is the same angle accounting for the presence of the hull. A default value of $\gamma = 1.0$ will be used if the user does not supply a value.

6.7.3 Implementation

Step 1 is performed before the start of the time stepping procedure. The remaining steps are performed at each time step. All of the steps will be performed separately for each propeller on the ship.

Step 1. Read input, compute time independent values

Prior to the start of the time stepping calculations the program will first read in all the user supplied inputs. A unit vector defining the direction of the propeller shaft in the ship-fixed reference frame will be defined using the formula:

$$\vec{n}_S = \begin{pmatrix} -\cos\beta_S \cos\alpha_S \\ \sin\beta_S \\ -\sin\alpha_S \cos\beta_S \end{pmatrix} \tag{6.110}$$

where, α_S describes the vertical inclination of the propeller shaft and is defined as the angle between the x-axis of the ship-fixed frame and the projection of the shaft axis onto the vertical plane of symmetry of the ship. A positive angle indicates that the shaft points downwards. β_S describes the orientation of the propeller shaft in the horizontal plane relative to the ship x-axis with a positive angle indicating that the shaft points to port.

Step 2. Compute velocity vector at center of propeller disk

At each time step the velocity vector at the center of the propeller disk is computed. The velocity is computed by adding the velocity from the motion of the ship to the wave orbital velocity. The procedure for computing the wave orbital velocity at an arbitrary point in space is described in a separate document. The computations for the wave orbital velocity are performed first in the earth-fixed frame, but must be transformed to the ship-fixed frame before being added to the velocities from the motion of the ship. The velocity vector at the center of the propeller disk (x_p,y_p,z_p) is computed as:

$$\vec{U}_P = -\begin{pmatrix} u + z_p q - y_p r \\ v + x_p r - z_p p \\ w + y_p p - x_p q \end{pmatrix} + \begin{pmatrix} u_w \\ v_w \\ w_w \end{pmatrix} , \tag{6.111}$$

where, (u,v,w) is the velocity of the ship defined at the ship-fixed origin, (p,q,r) is the rotational velocity of the ship, (x_p,y_p,z_p) is the position of the center of the propeller disk and (u_w,v_w,w_w) is the wave orbital velocity at the center of the propeller disk. All the above quantities are defined in the ship-fixed reference frame. The negative sign before the first vector in the formula for $\vec{U}_P$ indicates the inflow velocity into the propeller is in the opposite direction from the ship motion.

Step 3. Decompose velocities into components parallel and perpendicular to shaft. Compute cross flow angle and velocity.

The component of the velocity parallel to the propeller shaft is the dot product of the velocity vector with the unit vector parallel to the shaft axis:

$$U_A = \vec{U}_P \cdot \vec{n}_S = \cos\beta_S \cos\alpha_S (u + z_p q - y_p r) - \sin\beta_S (v + x_p r - z_p p) + \sin\alpha_S \cos\beta_S (w + y_p p - x_p q) \tag{6.112}$$

The cross flow velocity is then the difference between the total velocity vector and the component parallel to the shaft:

$$\vec{U}_{CROSS} = \vec{U}_P - U_A \vec{n}_S \tag{6.113}$$

The unit normal vector in the direction of the cross flow can be computed from $\vec{U}_{cross}$:

$$\vec{n}_C = \frac{\vec{U}_{CROSS}}{\left|\vec{U}_{CROSS}\right|} , \tag{6.114}$$

and the cross flow angle, ϕ_S, is defined as:

$$\phi_S = \arctan\left(\left|\vec{U}_{CROSS}\right| \big/ U_A \right) \tag{6.115}$$

ϕ_S should always be a positive angle between 0 and $\pi/2$. The unit vector defining the direction perpendicular to both the shaft axis and the cross flow is computed as the cross product of the unit vectors defining the shaft axis and cross flow directions:

$$\vec{n}_N = \vec{n}_S \times \vec{n}_C = \begin{pmatrix} n_{S2}n_{C3} - n_{S3}n_{C2} \\ n_{S3}n_{C1} - n_{S1}n_{C3} \\ n_{S1}n_{C2} - n_{S2}n_{C1} \end{pmatrix} \tag{6.116}$$

Defined in this way, $\vec{n}_N$ will point upwards when the cross flow is to starboard and point downwards when the cross flow is to port. The magnitude of the vector formed by the cross product of two vectors is defined as: $\left|\vec{a} \times \vec{b}\right| = |\vec{a}|\left|\vec{b}\right|\sin\theta$, where θ is the angle between the two vectors. In the case of $\vec{n}_S \times \vec{n}_C$, both vectors are unit vectors and they are perpendicular, so the angle between them should be 90°. They should also have a magnitude of the cross product vector equal to one. Computing the magnitude of this vector would provide a useful check.

Step 4. Compute the propeller thrust, side force and normal force

The propeller thrust is computed by interpolating the user supplied K_T vs. J. The advance coefficient J is first computed from the axial velocity adjusted to account for the presence of the hull using the Taylor wake fraction:

$$J = \frac{U_A(1 - w_F)}{n\,D} \tag{6.117}$$

The value for K_T is then obtained by interpolating the user supplied data. If the input data does not cover this range, it should be extrapolated to include these end points, with a warning printed to the output file. Negative values of J may also be specified to provide data for reverse flow into the propeller, where U_A is negative and n is positive. If the computed value for J is outside the range of user-supplied data for K_T vs. J, extrapolation should not be used, and the value used for K_T should remain constant outside the range of specified data. In the Level-0 implementation it is assumed that the revolutions per second, n, is a constant value input by the user. Later implementations may improve on this to account for the influence of the propeller loading on the rpm of the propeller. If the value for the rotational speed, n, is zero, then both J and K_T should be set to zero along with all the propeller forces. The K_T vs. J should cover values of J ranging from 0 to the value of J where the thrust is zero.

After computing K_T, the thrust is computed as:

$$T = K_T \rho\, n^2 D^4 \tag{6.118}$$

For cases where the U_A is very small, the cross flow angle, ϕ_S, as computed above may give values close to 90° even for small cross flow velocities. It would be expected that the induced velocity into the propeller would dominate the local flow in these cases. Therefore, after the thrust is computed, the velocity induced by the propeller at the center of the propeller disk will be computed from momentum theory as:

$$U_{A\_PROP} = \sqrt{\frac{4T}{\rho \pi D^2}} \tag{6.119}$$

If the value computed for U_{A_PROP} is greater than the value of U_A, then the cross flow angle, ϕ_S, will be recomputed replacing U_A with U_{A_PROP} in the formula listed in Step 3.

Next the side force and normal force are computed based on the empirical formulae derived in Fuhs and Dai (2007).

$$\begin{aligned} F_{CROSS} &= 1.716\, T \sin(\gamma \phi_S) \\ F_{NORM} &= 0.2\, F_{CROSS} \end{aligned} \tag{6.120}$$

where γ is an optional input value with a default value of 1.0 that may be used to account for the flow straightening influence of the hull.

Step 5. Compute submerged area of the propeller disk

The following is one way to calculate the submerged area of the propeller disk. The calculation of the submerged area of the propeller disk, A_S, is performed in the earth-fixed frame. The vertical position of the center of the propeller disk in the earth-fixed frame, Z_P^E, is first computed from (x_P, y_P, z_P) using the transformation formulae described in the Section 3.2. The "E" superscripts on variables in this section indicate that the values are defined in the earth-fixed frame. Z_W^E is the wave elevation directly above the propeller. It is assumed that the wave is long relative to the diameter of the propeller, so the wave height is assumed to be constant above the propeller. The inclination of the shaft is also ignored during the calculation of the submerged area. The total area of the propeller disk is: $A_D = \pi R^2$, where R is the propeller radius, ½ D.

If the value of Z_W^E is greater than $Z_P^E + R$, then the propeller is completely submerged and $A_S = A_D$. If the value of Z_W^E is less than $Z_P^E - R$, then the propeller is completely out of the water and $A_S = 0$. For all other conditions, the propeller is partially submerged and the submerged area must be computed. This calculation can be made by first computing the angle θ shown in Figure 6-7.

$$\theta = \arcsin\left(\frac{Z_W^E - Z_P^E}{R}\right) \tag{6.121}$$

The angle θ will be between $-\pi/2$ and $+\pi/2$. It will be positive if the wave is higher than the center of the propeller disk and negative if the wave is below the center of the propeller disk. The submerged area can now be computed in terms of θ and R as:

$$A_S = R^2\left(\cos\theta \sin\theta + \frac{\pi}{2} + \theta\right) \tag{6.122}$$

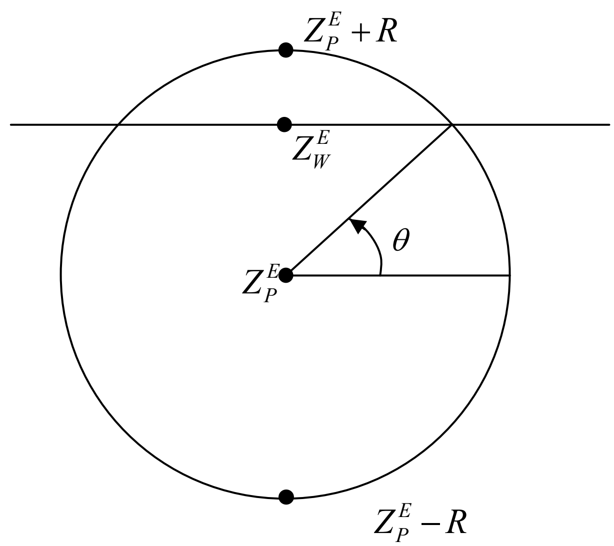

Figure 6-7 Calculation of submerged area of propeller disk

An alternate way to approximate the area is to panelize the propeller disk and trim the panels using the undisturbed free surface as a cutting surface. This is the approach taken by Tempest.

Step 6. Compute forces and moments in ship-fixed frame.

The thrust is in the direction of the shaft axis, but in the opposite direction from the unit vector defining the shaft axis direction, so the thrust is applied in the direction: $-\vec{n}_S$. The side force F_{CROSS} is in the direction of the cross flow $\vec{n}_C$ for both right hand turning and left hand turning propellers. The direction of the normal force, F_{NORM}, depends on both the direction of the cross flow and whether the propeller is right hand turning or left hand turning. For a right hand turning propeller, the normal force will point upwards if the cross flow is to port and downwards if the cross flow is to starboard. The normal force will point in the opposite direction for a left-handed propeller. The unit vector $\vec{n}_N$ is defined such that it points upwards when the flow is to starboard. Therefore the direction of the normal force can be defined by the vector: $-RHS\,\vec{n}_C$, where RHS is +1.0 for a right hand propeller and -1.0 for a left hand propeller.

The propeller force vector in the ship-fixed reference frame can be computed from the force components parallel and perpendicular to the propeller shaft:

$$\vec{F}_P = \frac{A_S}{A_D}\left(-T\,\vec{n}_S + F_{CROSS}\vec{n}_C - F_{CROSS}\ RHP\,\vec{n}_N\right)\begin{pmatrix}(1-t_p)\\ 1\\ 1\end{pmatrix} \tag{6.123}$$

The x component of the force in the ship-fixed frame is adjusted to account for the additional drag on the hull resulting from the interaction between the hull and propeller using the user specified thrust deduction fraction, t_P. All three components of the force are multiplied by the ratio of the submerged disk area to the total disk area: A_S / A_D .

The moments which result from the propeller force are computed from $\vec{F}_P$ and (x_p,y_p,z_p):

$$\vec{M}_P = \begin{pmatrix} F_{PZ} y_P - F_{PY} z_P \\ F_{PX} z_P - F_{PZ} x_P \\ F_{PY} x_P - F_{PX} y_P \end{pmatrix} \tag{6.124}$$

$\vec{F}_P$ and $\vec{M}_P$ are the force and moment from the propeller acting at the ship-fixed reference frame origin, which is the ship CG in Level-0. These two vectors are added to the other force components to compute the total force and moment action on the ship at each time step.

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